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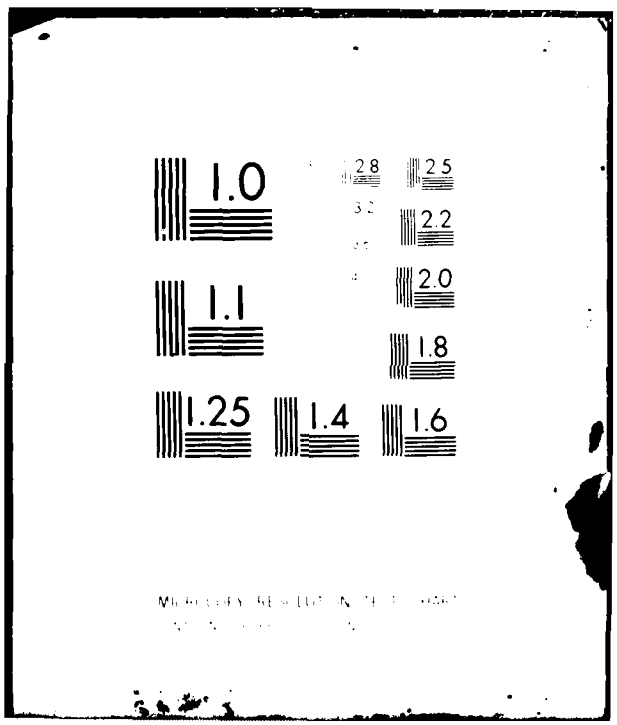
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CARRIER LOCALISATION IN INVERSION LAYERS AND IMPURITY BANDS

ANNUAL TECHNICAL REPORT

by

M. PEPPER

November 1981

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United States Army
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This report is principally concerned with our work on the physics of transport in two dimensional systems. We show that the logarithmic corrections to the conductance of Si inversion layers arise from both interaction and localization effects. Application of a magnetic field suppresses localization and enhances the role of interactions. At certain values of magnetic field both effects can be present, but with a different stability against increasing temperature. Consequently, heating the electron gas with an electric field allows

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the observation of a transition between them. Decreasing elastic scattering decreases the magnetic field required to suppress localization, as the enhancement of the interaction effect arises from spin a clear separation is obtained between the mechanisms.

The electron-electron scattering rate has been investigated and it is found that quantum corrections produce a deviation from the Landau-Baber T^2 law. We have also proposed the dependence of the conductance on a universal length.

Similar experiments are reported on GaAs and further work on the conductance oscillations are described. Standard commercial GaAs FET's exhibit the oscillations but the clear periodicity is not always present, possibly this is related to the "dirtiness" of the system.

19. contd.

conductance oscillations in GaAs

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This report is principally concerned with our work on the physics of transport in two dimensional systems. We show that the logarithmic corrections to the conductance of Si inversion layers arise from both interaction and localization effects. Application of a magnetic field suppresses localization and enhances the role of interactions. At certain values of magnetic field both effects can be present, but with a different stability against increasing temperature. Consequently, heating the electron gas with an electric field allows the observation of a transition between them. Decreasing elastic scattering decreases the magnetic field required to suppress localization, as the enhancement of the interaction effect arises from spin a clear separation is obtained between the mechanisms.

The electron-electron scattering rate has been investigated and it is found that quantum corrections produce a deviation from the Landau-Baber T^2 law. We have also proposed the dependence of the conductance on a universal length.

Similar experiments are reported on GaAs and further work on the conductance oscillations is described. Standard commercial GaAs FET's exhibit the oscillations but the clear periodicity is not always present, possibly this is related to the "dirtiness" of the system.

KEYWORDS

transport in semiconductors

two dimensional transport

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electron localisation

electron-electron scattering in Si and GaAs

conductance oscillations in GaAs

CONTENTS

Introduction

This report is the final technical report of this particular grant. During its term extensive progress was made in understanding two dimensional transport. The conductance oscillations were found and it was shown that ballistic injection into semiconductors was a very useful method of investigating phonons and inelastic scattering in semiconductors. It was also shown that spin dependent recombination could be applied to gate controlled Si p-n junctions and the g value of the centres was determined.

In this, the final report of the present grant, most attention is devoted to the problem of logarithmic corrections in two dimensional transport; essentially, the physics of this problem is now understood very well.

Localisation in Two Dimensional Systems

We have investigated the validity of theorems claiming that true diffusion does not occur in two dimensions (2D). It was initially suggested by Abrahams et al.[1] that all states in two dimensions are weakly localised. (The quantum analogy of Polya's[2] theorem (1921), that a random walker in two dimensions will always return to the origin as the system size is increased). The conclusion of Abrahams et al. has since been reached by other theorists[3,4]. In the weak scattering limit it is suggested that the conductance of a 2D system is given by

$$\sigma = \sigma_o - \frac{e^2\alpha}{\pi^2 h} \ell n(L/\ell) \quad . \tag{1}$$

Here σ_o is the normal conductance, $ne\tau/m^*$ (τ is the mean free time for scattering and m is the effective mass), α is a constant having a value of unity or half depending on the extent of spin scattering. L is the system size which is the specimen length at absolute zero, and at finite temperatures is the distance an electron diffuses before being scattered into another state. The inelastic scattering which is responsible for this process is electron-electron scattering which at low temperatures dominates over electron-phonon scattering. L can be defined as $L = (\frac{1}{2}\ell\, \ell_{IN})^{\frac{1}{2}}$ where ℓ and ℓ_{IN} are the elastic and inelastic mean free paths' respectively, $\ell = V_F\tau$ where V_F is the Fermi velocity.

As L will vary as T^p, where T is the temperature and p an appropriate power (p = 2 according to Landau-Baber theory) the conductivity correction becomes

$$\frac{e^2\alpha p}{2\pi^2\hbar} \ell n\, T \ . \tag{2}$$

Another theory which predicts a logarithmic temperature dependence was proposed by Altshuler, Aronov and Lee[5,6]. These authors had incorporated the effects of elastic scattering, and broadening of the electron energy, into Fermi liquid theory. They find a conductance correction, σ, given by

$$\sigma = \frac{e^2}{4\pi^2\hbar} (2 - 2F)\ell n\, T \tag{3}$$

where F is a factor arising from screening which is near unity for low

values of carrier concentration and tends to zero as the carrier concentration increases. Thus, if F is zero and $\alpha p = 1$, there is no difference between the predictions of the two theories, equations 3 and 2. However, the theories of the magneto-resistance and Hall effect indicate that here the behaviour will be very different in the two cases[5,6,7]. It is suggested that in the localization regime the Hall mobility will have the same temperature dependence as the conductance, i.e. decreasing logarithmically with temperature. On the other hand, for the interaction regime theory predicts that the Hall constant R_H varies as

$$\frac{\delta R_H}{R_H} = -\frac{2\delta\sigma}{\sigma} \qquad (4)$$

i.e. the carrier concentration decreases at twice the rate of the conductance, implying a Hall mobility which increases logarithmically as the temperature decreases. In the localisation regime the effect of a magnetic field is to decrease the scale length of the system, reducing the conductance correction and giving a negative magneto-resistance. Initially then it was thought that there was no magneto-resistance in the interaction regime, but this conclusion was subsequently modified.

The first experimental results on inversion layers and metal films indicated the existence of the logarithmic correction, but it was not clear which mechanism was responsible. The paper of Uren, Davies and Pepper showed that both mechanisms were present, the abstract of this paper is now reproduced.

"The observation of interaction and localisation effects in a two dimensional electron gas at low temperature",
- M.J. Uren, R.A. Davies and M. Pepper, J. Phys. C13, L985, 1980.

Abstract

We have investigated the logarithmic regime of transport of the two dimensional electron gas in the Si inversion layer between .05K and 1K. There was little band tailing in the specimens used, and it was possible to achieve the $k_F\ell \sim 1$ condition at carrier concentrations as low as $\sim 2.10^{15}$ m^{-2}. The behaviour of the conductance and Hall effect suggest that in the presence of a magnetic field the logarithmic corrections are caused by the electron-electron interaction. In particular, we confirm the prediction of this theory that the Hall coefficient varies at twice the rate of the resistance. However, at zero, or low values of, magnetic field it appears that the transport behaviour arises from localisation rather than interactions. At particular values of magnetic field these two mechanisms are both present, but have a different stability against an increase in temperature. Consequently, at a constant lattice temperature, it is possible to observe a transition between them as the electron temperature is increased with an electric field.

This paper resolved the problem by showing that both theories were correct, the essential feature of the inversion layer being that because the values of carrier concentration were low, $\sim 10^{12}$ cm^{-2}, F was $\sim$ 0.9. Consequently, the interaction mechanism was not observed in the absence of a magnetic field. It was subsequently theoretically shown that a magnetic field enhanced the interaction effect by converting (2 - 2F) in equation (3) into (2 - F).

The existence of the logarithmic correction, typically 7% per decade of temperature, allows the measurement of the electron temperature as a function of applied electric field. This is one of the few methods of finding the electron temperature of a metallic system. As Anderson and co-workers[8] have shown, plotting the dependence of applied electron temperature on electric field allows the determination of the temperature dependence of the rate of energy dissipation. This will give the dimensionality of the phonons emitted by the electrons. It was found that the emission rate $1/\tau$ varied as T_e, where T_e is the electron temperature. As this particular temperature dependence is not characteristic of phonons, it points to the possible existence of tunnelling centres at the interface. Absorption of energy by these centres will have the observed temperature dependence, the existence of these centres has also been invoked by other authors to explain hot electron phenomena at temperatures near 4.2 K.[9]

The effect of a magnetic field is to reduce the logarithmic correction. Physically this effect arises because the magnetic field introduces a new length scale - the cyclotron length. Essentially the mixing of states by the magnetic field cuts off the increasing definition of a state as the inelastic diffusion length increases. This problem has been considered in detail by Hikami et al.[10] and Altshuler et al.[6], these authors find that in the presence of a magnetic field, B, the conductance correction, $\delta\sigma$, is given by

$$\delta\sigma = \frac{e^2}{\pi^2\hbar}\left[\psi(\tfrac{1}{2} + \hbar/4eB\,\tau_{IN}\,D) + \ln(4eB\tau/\hbar)\right] . \qquad (5)$$

Here τ and τ_{IN} are the elastic and inelastic scattering times, D is the diffusivity of an electron ($V_F{}^2\tau$, where V_F is the Fermi velocity) and ψ is the Digamma function, also known as the Psi function. The Digamma function is rather unwieldy and lacks physical expression. As the localisation is only dependent on the length scales it should be possible to define a more physical, and composite length. We have invoked the length L_C which is given by

$$L_C{}^{-1} = (L_1{}^{-2} + L_2{}^{-2} + L_3{}^{-2} + L_4{}^{-2} \ldots)^{-1}.$$

L_1 etc. are the individual length scales which may be the inelastic diffusion length, cyclotron length, diffusion length in the presence of external radiation etc. The results of these considerations were published in -

"Localisation in disordered two dimensional systems and the universal dependence on diffusion length", - M. Kaveh, M.J. Uren, R.A. Davies and M. Pepper, J. Phys. C14, L413, 1981.

Abstract

We extend the treatment by Kaveh and Mott of the effect of weak localisation on the conductivity of disordered two-dimensional systems to include the effect of magnetic and electric fields on the localisation. It is shown that the change in conductivity due to the localisation depends on only one parameter, the diffusion length. This result holds irrespective of the number of independent mechanisms which individually correspond to a separate diffusion length. Our experimental values of the conductivity for different temperatures, electric fields and different magnetic fields fall on the predicted universal curve. The derived formulae for the conductivity, when few mechanisms determine the length scale, are in agreement with our

experimental data, which enables us to identify the individual diffusion lenghts.

The logarithmic correction due to localisation is a low temperature effect in Si inversion layers because electron-electron scattering causes a rapid transition between localised states and, hence, $L \sim \ell$ above the helium range of temperature. The normal electron-electron scattering law which is encountered in metals is that originally derived by Baber, in which $\tau_{e-e}^{-1} \propto T^2$. This law arises from scattering across the Fermi surface, and the T^2 comes from the product of the number of electrons which can be scattered varying as T and the number of states into which they can be scattered also varying as T. In the presence of impurity scattering, the electronic wavefunction is broadened over a range of energy near the Fermi energy. The effect of the broadening is to alter the rate of scattering when the change in electron momentum is small. In two dimensions this leads to a law of the form $\tau_{ee}^{-1} = AT^2 + BT$ - the quantum law of electron-electron scattering. In three dimensions, the T term is replaced by $T^{3/2}$. In order to investigate this effect the electron-electron scattering rate at low temperatures was found from the temperature dependence of the negative magneto-resistance.

"Magnetic delocalisation of a two-dimensional electron gas and the quantum law of electron-electron scattering", M.J. Uren, R.A. Davies, M. Kaveh and M. Pepper, J. Phys. C14, L395, 1981.

Abstract

We discuss the effect of a magnetic field on the weak localisation of a two-dimensional electron gas. It is shown that, due to quantum corrections the electron-electron relaxation time τ_{ee} varies with electron temperature T as $\tau_{ee}^{-1} = A_1T + A_2T^2$, in the temperature range 3 K - 0.1 K. This short τ_{ee} causes a rapid transition between states which are weakly localised and so reduces the logarithmic correction to the conductance.

In the previously quoted paper of Uren, Davies and Pepper it was shown that logarithmic corrections were present which arose from both localisation and interaction effects. Increasing the magnetic field suppressed localisation and enhanced interactions, but a logarithmic correction was always present. Increasing the mean free path for elastic scattering increases the diffusion length and so the localisation will be suppressed by a lower values of magnetic field. As the magnetic enhancement of the interaction effect arises from a spin effect, the magnetic field dependence of this mechanism should be unaffected.[11] Applying a parallel magnetic field should enhance the interaction effect without affecting localisation, thus the additive combination of the two will result.

These effects were explored in the following paper -

"Magnetic separation of localisation and interaction effects in a two-dimensional electron gas at low temperatures", R.A. Davies, M.J. Uren and M. Pepper, J. Phys. C14, L531, 1981.

Abstract

We show that by the application of a magnetic field it is possible

to achieve complete separation of localisation and interaction mechanisms in two dimensions. Measurements of the conductance of silicon inversion layers show that, at certain values of magnetic field, it is also possible to achieve metallic conduction near 50 mK. It is also shown that the appearance of the interaction mechanism is not strongly dependent on the direction of the magnetic field, implying that the origin of the effect is in electron spin rather than cyclotron orbit motion. The implications for the quantised Hall resistor are discussed.

The implications for the quantised Hall resistor[12] are clear. The localisation correction is suppressed by a magnetic field and interactions do not affect σ_{xy}. Thus, unless second order effects in localisation are insensitive to B, the quantisation of σ_{xy} is unafffected.

This type of study of two dimensional transport has been extended to the two dimensional gas in GaAs at the interface between undoped GaAs and doped GaAℓAs. It is known that in these structures electrons in the GaAℓAs fall into the GaAs and form a degenerate, two dimensional, electron gas at the interface of the two materials. Structures grown by Molecular Beam Epitaxy show very high values of electron mobility.[13] The samples used in our experiments were grown by liquid phase epitaxy and showed values of mobility, about 2.10^4 cm^2 volt^{-1} sec^{-1}. The specimens were not gated and the carrier concentration was $\sim$1.4 10^{12} cm^{-2}. It was found that both localisation and interaction effects were present, this latter effect being greater than in Si because of the greater 2D screening distance, and smaller values of F, in GaAs. Application of a magnetic field suppresses localisation and allows clear observation of the interaction mechanism. The electron-electron scattering rate was measured and found to deviate from the Landau-Baber T^2 law. Measurement of the rate of energy dissipation indicated that the phonons emitted by hot electrons were two dimensional, possibly an interfacial mode. This work was published in the following paper -

"The observation of localisation and interaction effects in the two-dimensional electron gas of a GaAs-GaAℓAs hetero-junction at low temperatures", D.A. Poole, M. Pepper and R.W. Glew, J. Phys. C14, L995, 1981.

Abstract

We have investigated the logarithmic correction to the transport properties of the two-dimensional electron gas at the (modulation-doped) GaAs-GaAℓAs interface in the temperature range 4.2 - 0.34 K. GaAs is different to Si in that, due to the low density of states, the electron screening length is greater. This allows the existence of a significant logarithmic correction from the electron-electron interaction in the absence of a magnetic field. The experimental results are consistent with the co-existence of localisation and interaction effects although the analysis is complicated by the occupation of two sub-bands.

We have investigated the electron-electron scattering rate and find that, as in the Si inversion layer, the temperature dependence is reduced by quantum corrections. Analysis of the rate of emission of phonons by hot electrons indicates that the phonons of importance are two-dimensional.

Conductance Oscillations in Two Dimensional Systems

We have pursued the oscillations found previously in GaAs structures.[14] In the initial experiments it was found that the minima occurred when the separation between electrons, r_e, was given by Nx where N is an integer and x is ∿110 A. As the temperature is lowered half and quarter integer minima appear. The original samples were n on p^+ structures and the conducting channel was forced away form the p^+ - n interface by the application of a substrate bias. We have now found that standard commercial devices show weak oscillations below ∿10 K. A number of such devices have been purchased and we are examining their characteristics. It often appears that the oscillations are linear with carrier concentration rather than with the separation of carriers. The cause of this is not clear but it may be related to the "dirtiness" of the systems.

Previously we had found that oscillations were produced in the accumulation regions over the source and drain regions of Si MOSFETS's.[15] In order to determine the relationship between the oscillations and carrier localization, change was induced in the oxide above an inversion layer by avalanch injection. After this process it was found that oscillations appeared in the conductance-gate voltage relation. At present it has not been possible to ascribe a periodicity to the oscillations, probably as a result of inhomogeneities. The temperature dependence of the conductance was measured and was found to be of the form $\sigma = \sigma_{min} \exp(-W/kT)$. Here σ_{min}, the minimum metallic conductance, was $\sim 310^{-5}\ \Omega^{-1}$ and the activation energy W oscillated as a function of carrier concentration. The exsitation to the mobility edge, indicates that the oscillations are not caused by electrons hopping between peaks in the density of states. Recent theoretical work suggests that the oscillations arise from electron ordering[16], and, if so, this would agree with the result that the excitation energy is an oscillating function of carrier concentration. However, final confirmation must await results on a cleaner inversion layer where a well defined periodicity is found.

References

1. Abrahams E, Anderson P.W., Licciordello D.C. and Ramakrishnan T.V., Phys. Rev. Lett., 42, 673, 1979.

2. Polya G., Math. Annalen, 84, 149, 1921.

3. Gorkov L.P., Larkin A.J. and Khmel'nitzkii D.E., J.E.T.P. Lett., 30, 229, 1979.

4. Haydock R.V., Phil. Mag., B43, 203, 1981.

5. Altshuler B.L., Aronov A.G. and Lee P.A., Phys. Rev. Lett., 44, 1288, 1980.

6. Altshuler B.L. and Aronov A.G., Solid State Comm., 36, 115, 1979.

7. Altshuler B.L., Khmel'nitzkii D., Larkin A.I. and Lee P.A., Phys. Rev., B22, 5142, 1980.

8. Anderson P.W., Abrahams E. and Ramakrishnan T.V., Phys. Rev. Lett., 43, 718, 1979.

9. Hönlein W. and Landwehr G., Surface Science, in the press, proceedings of the 1981 conference on Electronic Properties of Two Dimensional Systems.

10. Hikami S., Larkin A.I. and Nagaoka Y., Prog. Theor. Phys., 63, 707, 1980.

11. Lee P.A. and Ramakrishnan T.V., Phys. Rev. Lett., in the press.

12. Klitzing K., Dorda G. and Pepper M., Phys. Rev. Lett., 45, 494, 1980.

13. Drummond T.J., Kapp W and Morkoc H., Electron Lett., 17, 442, 1981.

14. Pepper M., J. Phys., C12, L617, 1979.

15. Pepper M., Uren M.J. and Oakley R.E., J. Phys., C12, L897, 1979.

16. Dallacasa V., J. Phys., C, in the press.

J. Phys. C: Solid St. Phys., 13 (1980) L985-93. Printed in Great Britain

LETTER TO THE EDITOR

The observation of interaction and localisation effects in a two-dimensional electron gas at low temperatures

M J Uren, R A Davies and M Pepper
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Received 4 September 1980

Abstract. We have investigated the logarithmic regime of transport of the two-dimensional electron gas in the Si inversion layer between 0·05 K and 1 K. There was little band tailing in the specimens used, and it was possible to achieve the $k_F l_{EL} \sim 1$ condition at carrier concentrations as low as $\sim 2 \times 10^{15}\ m^{-2}$. The behaviours of the conductance and the Hall effect suggest that in the presence of a magnetic field the logarithmic corrections are caused by electron–electron interaction. In particular, we confirm the prediction of this theory that the Hall coefficient varies at twice the rate of the resistance. However, at low values of magnetic field it appears that the transport behaviour arises from localisation rather than interactions. At particular values of magnetic field these two mechanisms are both present, but have a different stability against an increase in temperature. Consequently, at a constant lattice temperature it is possible to observe a transition between them as the electron temperature is increased with an electric field.

Recently there have been many contributions to the theory of electrical transport in two-dimensional (2D) systems. Early computer simulations of the 2D transport process (Licciardello and Thouless 1975) showed that a sharp mobility edge separated extended and localised states, and that there was a minimum metallic conductance σ_{min} with a value near $0{\cdot}1\ e^2/\hbar$ ($3 \times 10^{-5}\ \Omega^{-1}$). However, later calculations by these authors (Licciardello and Thouless 1978) indicated that σ_{min} decreased with increasing sample size. This suggested that, as the sample size went to infinity, σ_{min} went to zero, i.e. all states in 2D were localised. On the other hand, simulations by Lee (1979) failed to reveal this effect and he concluded that extended states, and a σ_{min} near $0{\cdot}1\ e^2/\hbar$, existed in 2D. All the calculations neglected the effects of the electron–electron interaction.

Abrahams *et al* (1979) (AALR) developed a one-electron scaling theory and also suggested that all electronic states are localised in 2D. These authors propose that when the sheet conductance σ of a system is $0{\cdot}1\ e^2/\hbar$, a transition occurs from strong to weak localisation. The weak localisation causes σ to behave as

$$\sigma = \sigma_0 - (e^2 \alpha/\pi^2 \hbar) \ln L \tag{1}$$

where σ_0 is the 'normal' conductance $ne^2\tau/m$, α is a constant with values $\frac{1}{2}$ or 1 and L is a scaling length. The meaning of L can be obtained by considering the Thouless (1974) expression for the conductance of a system:

$$\sigma = e^2\langle \Delta E\rangle / \hbar \langle \delta E\rangle. \tag{2}$$

Here $\langle \Delta E\rangle$ is the shift in energy of an average level when the boundary condition phase

is changed, and $\langle \delta E \rangle$ is the mean separation of adjacent levels. It is argued (Anderson *et al* 1979) that L, the effective sample size in equation (1), is the maximum diffusion length allowed before the smallest energy in equation (2) becomes indistinct due to inelastic scattering. Thus, if the inelastic scattering rate varies with temperature as T^b, L varies as $T^{-b/2}$ and the temperature-dependent logarithmic correction becomes

$$\Delta\sigma = (e^2 \alpha b / 2\pi^2 \hbar) \ln T. \qquad (3)$$

For electron–electron scattering $b = 2$, and $b = 2$, 3 or 4 for two-dimensional and three-dimensional phonon scattering in the dirty and clean limits respectively. AALR point out that, on their model, if $\Delta\sigma$ is erased by electron heating then a transition between activated and metallic conduction will occur when $\sigma = 0{\cdot}1\ e^2/\hbar$.

Fukuyama (1980a) suggests that equation (1) is not significantly affected by the valley degeneracy which exists in Si inversion layers. Abrahams and Ramakrishnan (1979, 1980) and Gor'kov *et al* (1979) have considered the quantum corrections in the impurity scattering of non-interacting electrons. They too find a logarithmic correction in the conductance. Haydock (1980) and Houghton *et al* (1980) have also suggested that all states in 2D are localised.

An alternative point of view has been proposed by Altshuler *et al* (1980a). They maintain that a logarithmic correction is not the result of localisation, but is produced by the electron–electron interaction in the presence of weak impurity scattering. In the low-frequency limit, they find the temperature-dependent correction is

$$\Delta\sigma = (e^2/4\hbar\pi^2)(2 - 2F) \ln T. \qquad (4)$$

F is determined by screening in the system, and if $k_F/K \ll 1$, where k_F is the Fermi k vector and K is the inverse 2D screening length,

$$F \sim 1 - 4k_F/\pi K. \qquad (5)$$

Fukuyama (1980b) has also obtained a similar result for interacting electrons.

Altshuler *et al* (1980b) (AKLL) have suggested that measurement of the Hall effect enables a distinction to be made between the localisation (AALR) and the interaction (Altshuler *et al* 1980a) models. Fukuyama (1980c) showed that the localisation model requires the Hall mobility to vary with temperature (or electric field) in the same manner as the conductance, i.e. the Hall constant R_H does not change. On the other hand, the interaction model predicts that the Hall conductance σ_{xy} is unaffected by the logarithmic correction in σ_{xx}. Thus, as $R_H = \sigma_{xy}/\sigma_{xx}^2$, the correction in R_H, ΔR_H, is (Altshuler *et al* 1980b)

$$\Delta R_H / R_H = -2\,\Delta\sigma/\sigma. \qquad (6)$$

Consequently the Hall mobility increases logarithmically with decreasing temperature.

The interaction model does not predict any magnetoresistance, whereas AKLL suggest that in the localisation model the magnetoresistance is negative and leads to the suppression of the logarithmic temperature dependence. This conclusion has also been drawn by Hikami *et al* (1980). The criterion for the suppression of the temperature dependence is that the cyclotron radius is comparable with the inelastic diffusion length. Below 0·1 K this condition can be achieved with fields of a few tens of Gauss.

A method of determining the inelastic scattering mechanism has been presented by Anderson *et al* (1979). For a constant lattice temperature, increasing the applied electric field E increases the electron temperature, and hence the conductance. If the electron–phonon relaxation time varies as T^{-p}, and if the inelastic scattering mechanism determining the behaviour of $\Delta\sigma$ as a function of temperature at low fields is unaffected by the

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field, then the ratio of the change in $\Delta\sigma$ as a function of $\ln T$ to the change as a function of $\ln E$ is $1 + p/2$. From simple phase space considerations we would not expect this formula to apply if $\Delta\sigma$, as a function of $\ln T$, is determined by electron-electron scattering. Here b would not equal 2 and might be expected to become a complicated function of E as this increases. In this context we note that it has been suggested that $b = 1$ for 2D electron-electron scattering in the presence of impurity scattering (Abrahams *et al* 1980a).

The first experimental observation of a $\ln T$ dependence was in thin metal films (Dolan and Osheroff 1979). A very weak decrease in extended state conductance in Si inversion layers is present in the results of Adkins *et al* (1976; figure 3). Recent work by Bishop *et al* (1980) (BTD) established that the decrease in conductance of Si inversion layers, for $k_F l_{EL} > 1$, below 1 K is consistent with the proposed $\ln T$ laws. BTD found $\alpha = \frac{1}{3}$ and $b = p = 3$; this value of α is too small to be satisfactorily explained by the localisation model.

A negative magnetoresistance is consistently observed in the logarithmic regime. Kawaguchi and Kawaji (1980) have fitted their results to the expression of Hikami *et al* (1980). However, their extraction of an inelastic scattering rate due to electron-electron collisions required the unphysical behaviour of a temperature-dependent α.

It is to be noted that in general the changes in σ are $\lesssim 10\%$, and the variation in T is a factor of 20: 1 K–50 mK. Consequently a logarithmic law cannot be inferred unambiguously; for example, compare this change in $\Delta\sigma$ to the three orders of magnitude variation necessary for the confirmation of the 2D hopping law (Mott *et al* 1975).

The experiments reported here were carried out on n-channel silicon gate MOSFETs fabricated on the (100) Si surface. The sample size was 250 μm × 250 μm and two pairs of potential probes were equally spaced along the channel. The threshold voltage of the devices was adjusted in the positive direction by the implantation of boron acceptors. Scattering from these charged acceptors limited the peak mobility at 4·2 K to values less than 0·65 $m^2\ V^{-1}\ s^{-1}$, depending on the doping. However, of more importance to this work was the fact that there was little band tail localisation (~ 2–4×10^{15} states m^{-2}). Thus measurements of the logarithmic correction could start at this value of carrier concentration, considerably lower than in the BTD experiments.

The samples were mounted onto a copper rod and connected to the mixing chamber of a dilution refrigerator. The temperatures were in the range 50 mK–1 K and were measured with a calibrated germanium thermometer. In order to ensure ohmic behaviour at the lowest temperatures an electric field of $(10$–$50) \times 10^{-3}\ V\ m^{-1}$ was necessary. The measurements were taken by conventional four-terminal techniques, using lock-in methods at low frequency (10 Hz). The samples did not normally show contact resistance, but the four-terminal technique was used to remove any residual effects.

We found the well known $\exp(T_0/T)^{1/3}$ hopping behaviour for sample resistances $R_\square$ greater than 10 kΩ, a factor of 4 lower than $10h/e^2$. When $R_\square$ was smaller than 10 kΩ a weak temperature dependence was found, consistent with a logarithmic law. A similar behaviour was found as a function of electric field at a constant temperature. Because the change in sample resistance in the logarithmic regime was small, the electric field dependence of the resistance was investigated by measuring the differential as the DC field was swept. Subsequent integration gave the change in resistance with improved accuracy.

The Hall effect was measured as a function of both electric field and temperature. The magnetic field employed was in the range 0·1–2·5 T. The Hall angle was normally less than 10°, the higher fields being used when the mobility was low. Figure 1(*a*) shows the behaviour of R_H as a function of E near 50 mK; the differential of V_H as a function of

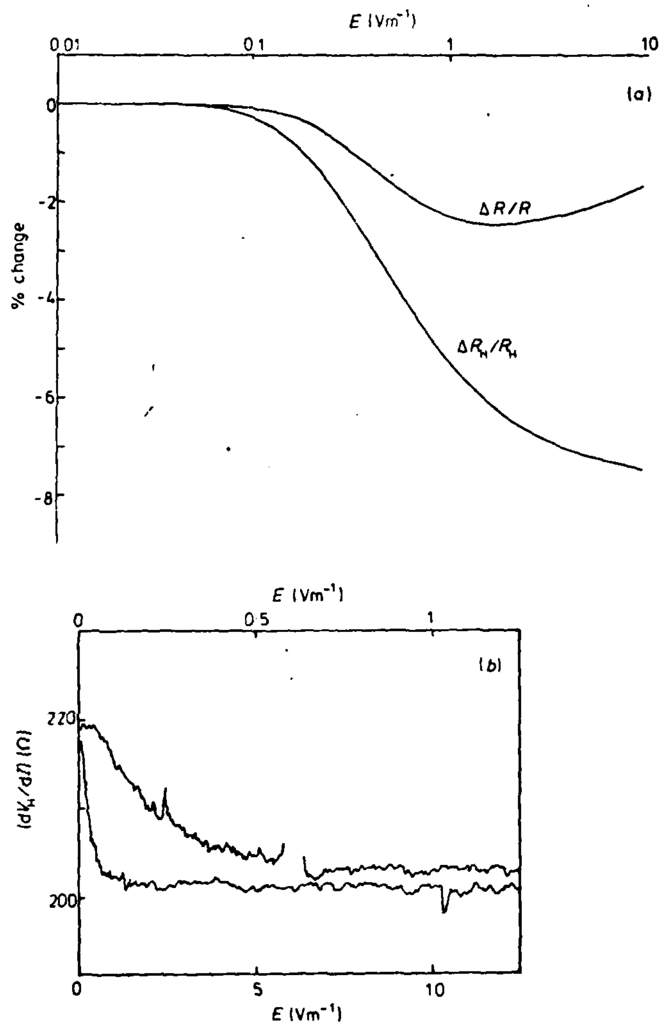
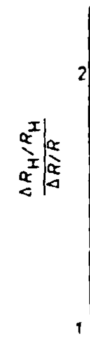


Figure 1. (*a*) Percentage changes in resistance R and Hall constant R_H against electric field. Doping $2{\cdot}3 \times 10^{22}\ \mathrm{m}^{-2}$; $B = 0{\cdot}4\ \mathrm{T}$; $R \simeq 1{\cdot}7\ \mathrm{k}\Omega$; $T = 50\ \mathrm{mK}$; $N_{inv} = 6{\cdot}1 \times 10^{15}\ \mathrm{m}^{-2}$. (*b*) Differential of the Hall voltage in figure 1(*a*) against two ranges in electric field. The break in the upper curve is a transient. The scale on the top refers to the upper tracing.

electric field is shown in figure 1(*b*). V_H is clearly seen to return to classical constant behaviour at high electric fields, and presumably, high electron temperature.

The ratio of the proportional change in Hall constant to the proportional change in resistance is shown in figure 2; these results were obtained with two samples of different doping. A ratio of 2 is found which persists as the carrier concentration N_{inv} is decreased. When N_{inv} is close to the onset of $T^{-1/3}$ behaviour, a sudden drop in value from 2 to 1 occurs for a change of N_{inv} of only $4 \times 10^{14}\ \mathrm{m}^{-2}$ in the lightly doped specimen. The drop from 2 to 1 is gradual in the more highly doped (lower mobility) sample. The ratio of 2 was found for all fields in the range 0·1–2·5 T.



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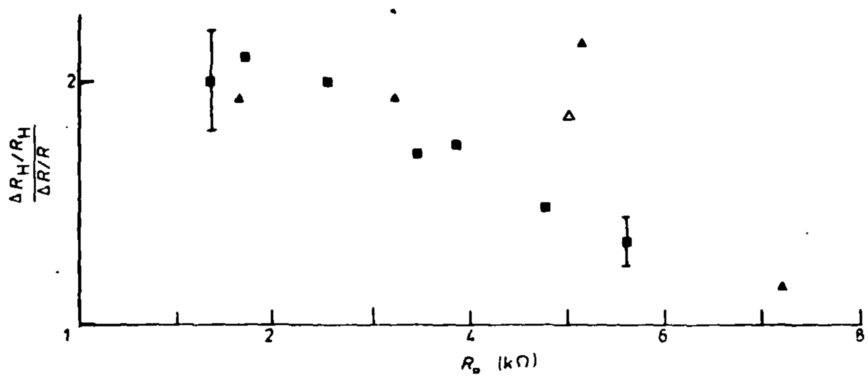

Figure 2. The ratio of the change in Hall constant to the change in resistance is plotted as a function of resistance. For the low doping device (triangles) $B = 0{\cdot}4$ T, $N_{inv} = 2{\cdot}5 \times 10^{15}$ m^{-2} for $R_\square = 7{\cdot}5$ kΩ and $2{\cdot}9 \times 10^{15}$ for $R_\square = 5$ kΩ. This corresponds to a field effect mobility of approximately 1 $m^2\,V^{-1}\,s^{-1}$ for $N_{inv} = 2{\cdot}7 \times 10^{15}$. For the high doping device (squares) $B = 1{\cdot}3$ T and N_{inv} varies from $7{\cdot}64 \times 10^{15}$ to $2{\cdot}34 \times 10^{16}$ m^{-2}. Full symbols, slopes versus E; open symbols, slopes versus T.

The low-temperature resistance was investigated as a function of magnetic field B. At 85 mK, increasing B caused a progressive decrease in the slope of the resistance versus ln E relation. However, as B increased, it appeared as if a second logarithmic

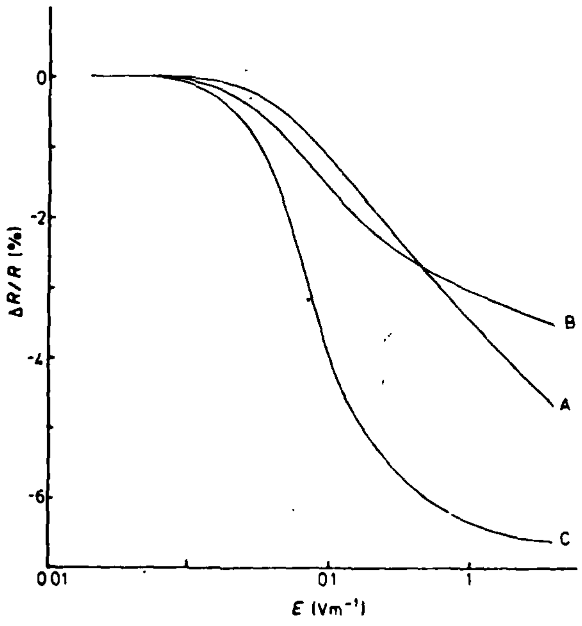

Figure 3. Percentage changes in the resistance against electric field for various magnetic fields B, showing the transition between two regimes. $N_A = 1{\cdot}8 \times 10^{23}$ m^{-3}, $N_{inv} = 9{\cdot}9 \times 10^{15}$ m^{-2}, $T \approx 85$ mK. A, $B = 0{\cdot}053$ T, $R_\square = 3{\cdot}88$ kΩ; B, $B = 0{\cdot}131$ T, $R_\square = 3{\cdot}79$ kΩ; C, $B = 0{\cdot}525$ T, $R_\square = 3{\cdot}73$ kΩ.

process was becoming operative. This new 'high B' process dominated when E was small, and as E increased a clear break between the two processes could be observed. This is shown in figure 3. Further increase in B resulted in the 'high B' process gradually extending over an increased range of electric field, until the first mechanism was not observable. At 85 mK the disappearance of the first process occurred at ~ 0·25 T.

When the two mechanisms were weak and of a similar magnitude (~ 0·1 T at 85 mK), it was difficult to determine if the change in $R_{\square}$ was logarithmic, with a consequent error on the slope. The gradients for the two mechanisms are plotted in figure 4, and we believe that when the 'high B' process is logarithmic the slope is in fact a constant, independent of B.

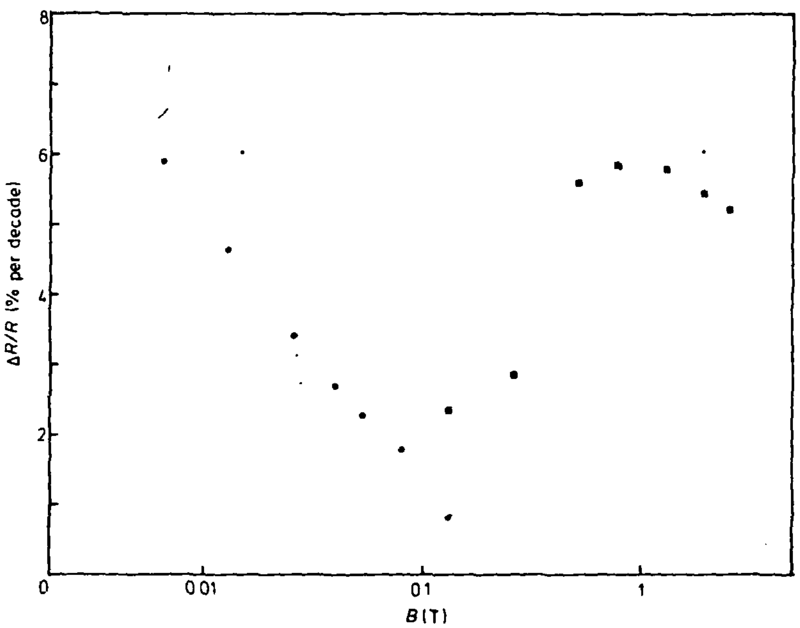

Figure 4. Percentage change in resistance per decade change in electric field for the same device and conditions as in figure 3. The two points at B = 0·131 T correspond to the two regions shown in figure 3 (curve B). Our separation of the two regimes is indicated by the labelling ●, ■.

Unfortunately, our Hall data became ambiguous at those values of B or E required for the first process to become dominant. It is not possible for us to say whether or not the ratio of 2 was maintained in this regime.

The temperature dependence of the resistance in the ohmic range was investigated. In figure 5 we show the temperature dependence for a field of 0·4 T and also when only a residual field was present, ($B \lesssim$ 0·005 T). As we can see, the 'high B' process, which is completely dominant at 0·4 T, only starts when the temperature is reduced to 0·3 K, whereas the first process (low B) is observed when the temperature is still above 1 K.

The resistance changes per decade of temperature and electric fields are shown in figure 6 for a constant magnetic field of 1·31 T. The results as a function of electric field are virtually identical to the zero-field results of BTD. However, the changes in $\Delta R/R$

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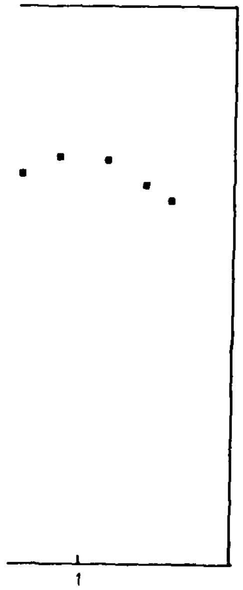

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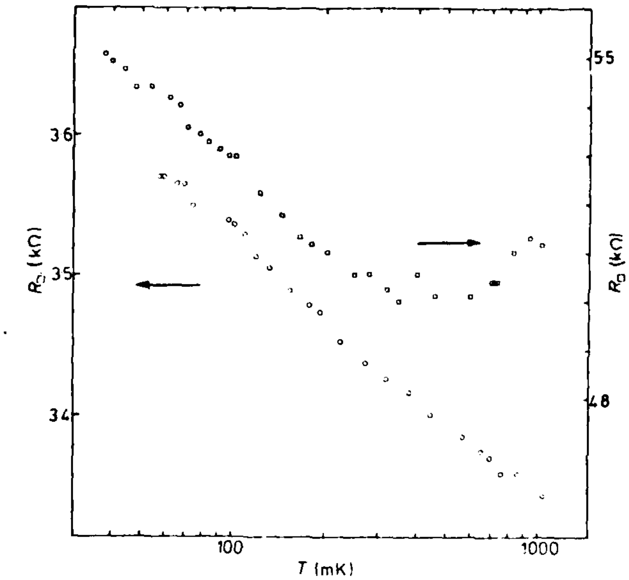

Figure 5. The change in inversion layer resistance against lg T. The scale on the right is for $N_{inv} = 2{\cdot}9 \times 10^{15}\ m^{-2}$, $B = 0{\cdot}4$ T. The scale on the left is for $N_{inv} = 3{\cdot}8 \times 10^{15}\ m^{-2}$ with only residual magnetic fields. ($N_A = 2{\cdot}3 \times 10^{22}\ m^{-3}$ for both).

as a function of temperature differ considerably from BTD. Application of the heating model of Anderson *et al* gives a value of xb of $\simeq 1$ and a value of p of $\simeq 1$.

Turning now to a discussion of the results, the ratio of 2 found for the ratio of the

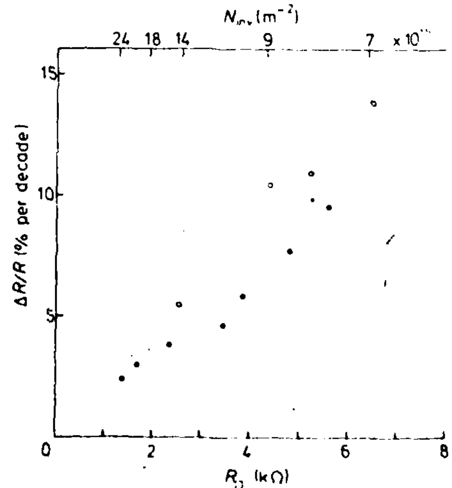

Figure 6. Change in resistance per decade change in electric field or temperature against $R_\square$. $B = 1{\cdot}31$ T; N_{inv} is indicated on the upper scale. $N_A = 1{\cdot}8 \times 10^{23}\ m^{-3}$. Open circles, slopes versus T; full circles, slopes versus E.

proportional changes of R_H and resistance is good evidence for the validity of the interaction model, i.e. the 'high B' process. It must be born in mind that this is only true for $B \geqslant 0{\cdot}1$ T, as reliable Hall data were not obtained at smaller B. We can therefore compare the magnitude of the logarithmic slope of the 'high B' process with the predictions of Altshuler *et al* (1980a). The range of carrier concentration was such that $0{\cdot}076 < k_F/K < 0{\cdot}2$; thus the factor F (defined by equation (5)), is in the range 0·75–0·9. Equation (4) is modified in the presence of a magnetic field, the factor $2 - 2F$ being replaced by $2 - F$ (AKLL, Appendix B). Hence the logarithmic correction now becomes

$$\Delta\sigma = C(e^2/\pi^2\hbar) \ln T. \tag{7}$$

The numerical constant C increases from 0·56 to 0·63 for the values of N_{inv}, and k_F, used in these experiments. The experimental value of C is $0{\cdot}77 \pm 0{\cdot}02$, which is in reasonable agreement with theory, particularly bearing in mind the general uncertainty about screening. In the absence of a magnetic field C is considerably reduced and this may account for the absence of these mechanisms at low B. It is to be noted that the value of C of $0{\cdot}77 \pm 0{\cdot}02$ is lower than the value predicted by localisation theory if $\alpha = 1$, i.e. a long spin flip scattering length, expected in Si where the spin–orbit coupling is weak.

As previously pointed out, the application of the heating model to the data of figure 6 yields an electron–phonon scattering rate proportional to T, i.e. $p \sim 1$. This does not correspond to any known mechanism and indicates that the heating model is invalid. We expect this if the temperature dependence of the ohmic resistance is determined by electron–electron scattering, or a general electron–electron interaction as implied by the Hall ratio of 2. Thus, there appears to be convincing evidence that 'high B' mechanism is the electron–electron interaction process postulated by Altshuler *et al* (1980a). This model also predicts an absence of magnetoresistance, which is observed.

The Hall ratio of 2 is strictly valid only when $k_F l_{EI} \gg 1$ (l_{EL} is the elastic scattering length). This may account for the sharp drop in the ratio from 2 to 1 near the onset of $T^{-1/3}$ behaviour for the higher mobility (lightly doped) sample, whereas the ratio decreased more gradually for the lower-mobility sample with small l_{EL} (figure 2).

We now consider the process causing the logarithmic behaviour at low and zero B. The negative magnetoresistance in this regime and suppression of the logarithmic variation by small B are both in agreement with the predictions of the localisation theory. It therefore appears that we have evidence for the existence of both the localisation and the interaction mechanisms, the first being suppressed, and the second brought out, by the application of a magnetic field. Although the logarithmic gradient in the interacting state appeared to be independent of B, the range of electric field over which the change was logarithmic did increase with B. This appears to indicate that the interacting state is stabilised by B and can exist at higher electron temperatures as B increases.

Final confirmation of the existence of the two mechanisms must await measurements of the Hall effect in the localisation regime.

We are grateful to Professor Sir Nevill Mott, J H Davies, R V Haydock, M Kaveh and M J Kelly for many discussions on this topic. The low-temperature experiments were performed at the SRC Rutherford Laboratory and would not have been possible without the advice and help of Dr S F J Read and Mr G Regan. This work was supported by SRC and in part by the European Research Office of the US Army. M J Uren and R A Davies thank SRC for research studentships. We also thank V A Browne and R E Oakley of the Plessey Company, Allen Clark Research Centre for advice on specimen

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Note added in proof. We thank Drs Bishop, Tsui and Dynes for a preprint of their recent work on the behaviour of the Hall effect at magnetic fields greater than 0·1 T. They conclude that the behaviour of R_{xy} is in agreement with the prediction of Altshuler *et al.* However, as the dependence of the logarithmic slope was not investigated in detail at low fields, they conclude that only the interaction mechanism is operative. The results shown here demonstrate that in fact these are two mechanisms which can produce a logarithmic correction.

References

Abrahams E, Anderson P W, Lee P A and Ramakrishnan T V 1980 to be published
Abrahams E, Anderson P W, Licciardello D C and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **42** 673
Abrahams E and Ramakrishnan T V 1979 *Amorphous Materials Conference, Cambridge, Mass.*
—— 1980 *J. Non-Cryst. Solids* **35** 15
Adkins C J, Pollitt S and Pepper M 1976 *J. Physique* **37** C4 343
Altshuler B L, Aronov A G and Lee P A 1980a *Phys. Rev. Lett.* **44** 1288
Altshuler B L, Khmel'nitzkii D, Larkin A I and Lee P A 1980b to be published
Anderson P W, Abrahams E and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **43** 718
Bishop D J, Tsui D C and Dynes R C 1980 *Phys. Rev. Lett.* **44** 1153
Dolan G J and Osheroff D D 1979 *Phys. Rev. Lett.* **43** 721
Fukuyama H 1980a *J. Phys. Soc. Japan* **49** 649
—— 1980b *J. Phys. Soc. Japan* in press
—— 1980c *J. Phys. Soc. Japan* **49** 644
Gor'kov L P, Larkin A I and Khmel'nitzkii D E 1979 *JETP Lett.* **30** 229
Haydock R V 1980 *Phil. Mag.* in press
Hikami S, Larkin A I and Nagaoka Y 1980 *Prog. Theor. Phys.* **63** 707
Houghton A, Jericki A, Kenway R D and Pruisken A M M 1980 *Phys. Rev. Lett.* **45** 394
Kawaguchi Y and Kawaji S 1980 *J. Phys. Soc. Japan* **48** 699
Lee P A 1979 *Phys. Rev. Lett.* **42** 1492
Licciardello D C and Thouless D J 1975 *J. Phys. C: Solid St. Phys.* **8** 4157
—— 1978 *J. Phys. C: Solid St. Phys.* **11** 925
Mott N F, Pepper M, Pollitt S, Wallis R H and Adkins C J 1975 *Proc. R. Soc.* **A345** 169
Thouless D J 1974 *Phys. Rep.* **13** 93

J. Phys. C: Solid State Phys., 14 (1981) 413–9. Printed in Great Britain

LETTER TO THE EDITOR

Localisation in disordered two-dimensional systems and the universal dependence on diffusion length

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Abstract. We extend the treatment by Kaveh and Mott of the effect of weak localisation on the conductivity of disordered two-dimensional systems to include the effect of magnetic and electric fields on the localisation. It is shown that the change in conductivity due to the localisation depends on only one parameter, the diffusion length. This result holds irrespective of the number of independent mechanisms which individually correspond to a separate diffusion length. Our experimental values of the conductivity for different temperatures, electric fields and different magnetic fields fall on the predicted universal curve. The derived formulae for the conductivity, when few mechanisms determine the length scale, are in agreement with our experimental data, which enables us to identify the individual diffusion lengths.

It has been suggested (Abrahams *et al* 1979, Gorkov *et al* 1979, Houghton *et al* 1980, Haydock 1981, Kaveh and Mott 1981a, b) that all states are weakly localised in a disordered two-dimensional system. The nature of the localisation was recently studied by Kaveh and Mott (1981a). They described the random walk of an electron in the disordered material, i.e. suffering from elastic scattering, by a diffusion equation. The solution of the diffusion equation was correlated to the functional form of the wavefunction. The analysis showed that the amplitude of the wavefunction falls off as $1/r$ and it is essentially extended in the sense that it is not normalisable. At zero temperature they found the result of Abrahams *et al* (1979) that the metallic conductance decreases as ln L, where L is the size of the sample. At finite temperatures the electron remains in the same energy only over an inelastic scattering time τ_{in}. The elastic random walk of the electron is therefore described by the elastic diffusion equation only when $r < L_{in}(L_{in} = (\frac{1}{2}ll_{in})^{1/2} = (D\tau_{in}^{1})^{1/2})$, where l and l_{in} are the elastic and inelastic mean free paths respectively. This leads exactly to the result predicted by Anderson *et al* (1979):

$$\delta\sigma = -\frac{e^2}{\pi^2\hbar}\ln(L_{in}/l). \tag{1}$$

The success of the diffusion equation in reproducing in a simple way (Kaveh and Mott 1981a) the perturbation result obtained by Abrahams *et al* (1979) and Gorkov *et al* (1979) encouraged us to try and unify the effect of *all* possible external perturbations (such as

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electric and magnetic fields) on $\delta\sigma$ via a modification of the diffusion equation from a homogeneous equation to an *inhomogeneous* equation.

In this Letter we show that including external perturbations in the diffusion equation leads to a universal dependence of $\delta\sigma$ on one diffusion length. Our treatment supports the scaling idea of Abrahams *et al* (1979) in the metallic region, where $k_F l > 1$ (above a 'mobility edge'), that here only a length scale determines the conductivity.

The negative magnetoresistance predicted by Hikami *et al* (1980) and Altshuler *et al* (1980) follows naturally and in a physically clear manner from our treatment. We demonstrate here that the dependence of $\delta\sigma$ on the sample length, on the temperature, on the frequency, on the magnetic field and on the electric field shrinks into a universal curve which is described by only one parameter: the diffusion length scale L_d. Our experimental results confirm the hypothesis of universality and are shown in figure 1.

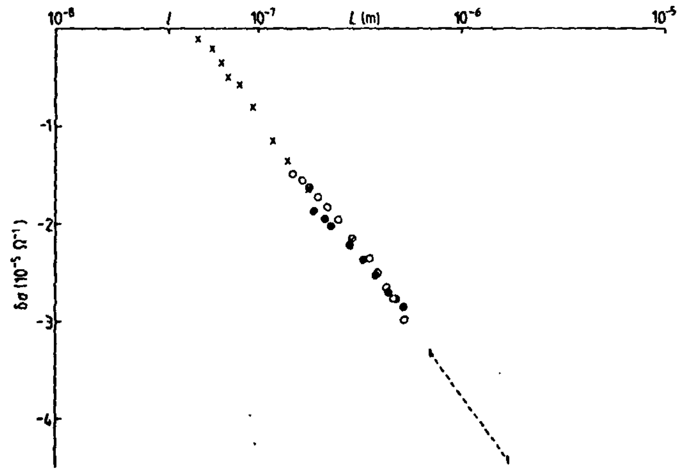

Figure 1. The change in conductivity is plotted against the log of the shortest length in the system. The crosses indicate the conductivity determined by magnetic field and so the length plotted is the cyclotron radius. Open circles indicate that the length is determined by $L_{in}(T)$ for ohmic electric fields and temperatures varying between 100 and 600 mK. Full circles indicate that $L_{in}(F)$ is the shortest length, which is changed by varying the electric field from 0.2–2.0 V m^{-1}. The broken curve indicates where L_F would lie for the same range of electric fields. However, the values of D are such that due to heating L_{in} becomes much shorter than L_F for the particular values of electric field.

Here all our data points for different temperatures, electric fields and magnetic fields fall on a universal curve which will be discussed in more detail later. In addition, we present an expression for the dependence of the conductivity correction on every individual diffusion length when more than one mechanism is operative. The dependence of σ on every individual diffusion length is verified experimentally, and excellent agreement is found between our predictions and the experimental results.

We first extend the treatment of Kaveh and Mott (1981a) to include external perturbations such as electric and magnetic fields. The random walk of the electron for

distances $r > l$ is not now given by a homogeneous diffusion equation. We may write

$$D\nabla^2 G(r, t) - \partial G/\partial t = [\partial G/\partial t]_{\text{ext}} \quad (r \geq l) \tag{2}$$

where $[\partial G/\partial t]_{\text{ext}}$ is the change in the probability of finding an electron at r and time t due to any source other than elastic scattering. Equation (2) is valid only for $r > l$, since no elastic scattering occurs when $r < l$. It is assumed that l is the shortest length in the problem and any diffusion length L_d (which will be determined from $[\partial G/\partial t]_{\text{ext}}$) is larger than l. If $[\partial G/\partial t]_{\text{ext}}$ produces a length $L_d < l$, the singular behaviour of $\delta\sigma$ is removed and there is no region in space in which the logarithmic correction to the conductivity, which is caused by the LHS of equation (2), takes place. In this case the conductivity will be completely metallic.

If the external perturbation persists for a period of time τ_p we may write

$$[\partial G/\partial t]_{\text{ext}} = G/\tau_p. \tag{3}$$

If we have few perturbations acting simultaneously, we may make the approximation $[\partial G/\partial t]_{\text{ext}} = G/\tau_{\text{eff}}$, where

$$\tau_{\text{eff}}^{-1} = \sum_p \tau_p^{-1}. \tag{4}$$

We now generalise the treatment of Kaveh and Mott (1981a) to include external perturbations; taking the Fourier transform of equation (2) we obtain

$$G(q, \Omega) = \frac{1}{Dq^2 + i\Omega + \tau_{\text{eff}}^{-1}}. \tag{5}$$

The amplitude of the wavefunction $\varphi(r)$ is correlated to $G(q, \Omega = 0)$ (Kaveh and Mott 1981a) by

$$\varphi(r) \propto \int dqq \exp(iq \cdot r)\, G(q, \Omega = 0). \tag{6}$$

This yields

$$\varphi(r) = (A/r) \exp(-r/L_d) \exp(iK \cdot r) \tag{7}$$

which is similar to equation (25) of Kaveh and Mott (1981a) except that we now have a *general* L_d which is given by

$$L_d = (D\tau_{\text{eff}})^{1/2}. \tag{8}$$

The conductivity is then

$$\sigma = \sigma_0 - (e^2/\pi^2\hbar) \ln(L_d/l). \tag{9}$$

This suggests that $\delta\sigma$ depends only on one parameter, the diffusion length scale L_d. The source of L_d does not matter. Even if several mechanisms operate simultaneously, equation (9) is still valid with the following law:

$$L_d = \left(\sum_p L_p^{-2}\right)^{-1/2} \tag{10}$$

and

$$L_d = (D\tau_p)^{1/2}.$$

We now show that equations (9) and (10) represent not only a scattering mechanism but

also external fields. We first discuss the negative magnetoresistance discussed by Hikami *et al* (1980) and Altshuler *et al* (1980). They found

$$\delta\sigma = \frac{e^2}{\pi^2\hbar}\left[\psi\left(\tfrac{1}{2} + \frac{\hbar}{4eB\tau_{in}D}\right) + \ln\left(\frac{4eB\tau}{\hbar}\right)\right]. \tag{11}$$

This expression seems to lie outside the universality idea of a unique diffusion length. However, the limiting form of equation (11) supports this idea: thus when $B \to 0$, $\delta\sigma \to (e^2/\pi^2\hbar)\ \ln(L_{in}/l)$, where $L_{in} = (D\tau_{in})^{1/2}$, and as $B \to \infty$, $\delta\sigma \to (e^2/\pi^2\hbar)\ \ln(L_c/l)$, where L_c is the cyclotron magnetic length, $L_c = (\hbar/eB)^{1/2}$. We therefore propose that the complicated behaviour of the effect of the magnetic field is a consequence of a competition between two mechanisms represented by two lengths L_{in} and L_c. The simple form of equation (9) is apparent only if one length is much shorter than the other. The complexity of the effect of a magnetic field arises in the form one should take for $[\partial G/\partial t]_B$ in equation (2). The inelastic scattering is simply represented by our relaxation time approximation, namely $[\partial G/\partial t]_{in} = G/\tau_{in}$. For simplicity, let us approximate the effect of the magnetic field by

$$[\partial G/\partial t]_B = G\omega_c. \tag{12}$$

If $L_B \ll L_{in}$, where $L_B = (D/\omega_c)^{1/2}$, the wavefunction according to equation (7) will be

$$\psi = (A/r)\ \exp(-r/L_B)\ \exp(\mathrm{i}K \cdot r). \tag{13}$$

However, even in the absence of diffusion due to elastic scattering, the wavefunction ψ is not extended and can be written as

$$\psi \sim \exp(\mathrm{i}K \cdot r)\ \exp[-\tfrac{1}{2}(r/L_c)^2]. \tag{14}$$

Usually the modulation of ψ over a distance larger than L_c is not important and the metallic conduction is observed. However, here in two dimensions, L_c can be the normalisation length of equation (13) if $L_c < L_B$. Indeed this is always the case, since

$$L_B = (\tfrac{1}{2}K_F l)^{1/2} L_c \tag{15}$$

and $K_F l \gg 1$, so that $L_B > L_c$. This gives $\delta\sigma \sim \ln(L_c/l)$ according to Kaveh and Mott (1981a), L_c being the normalisation length in equation (13) rather than L_B. This argument explains why the digamma function extrapolates for large B to $\ln(L_c/l)$ and not to $\ln(L_B/l)$. Thus the effect of a magnetic field is unique in that it introduces a length scale which is independent of the diffusion constant D.

In general,

$$\delta\sigma(T, B) = (e^2/\pi^2\hbar)\ \ln[L_d(T, B)/l] \tag{16}$$

where

$$L_d(T, B) = (L_{in}^{-2}(T) + L_c^{-2}(B))^{-1/2}. \tag{17}$$

This unifies the effect of the magnetic field to the idea of a one-scale length function. This also simplifies the result of Hikami *et al* (1980) and Altshuler *et al* (1980) by approximating their digamma (ψ) function into a simple length dependence. In figure 2, expressions (11) and (16) are compared as a function of the ratio L_{in}/L_c, for constant L_c. Equations (11) and (16) are normalised in the regime of L_{in} dominating and L_c dominating respectively. The important point is that the shapes are very similar in the region where $L_c \sim L_{in}$, and we find a temperature-dependent magnetoresistance. The effect of the constants is noted in the caption to figure 2.

We now turn to the effect of a static electric field. The effect of this field can be generalised by an argument developed by Kaveh and Mott (1981c) in their discussion of the validity of the scaling theory. The electron always undergoes a random walk in the presence of an electric field F. The change in energy of the electron in travelling the diffusion length L varies between $\pm eFL$. When this energy is less than the diffusion energy $\hbar D/L^2$ it can be neglected. This is always the case for small electric fields and large inelastic scattering rates (which cause a small diffusion length). However, a critical length L_F is always found for which

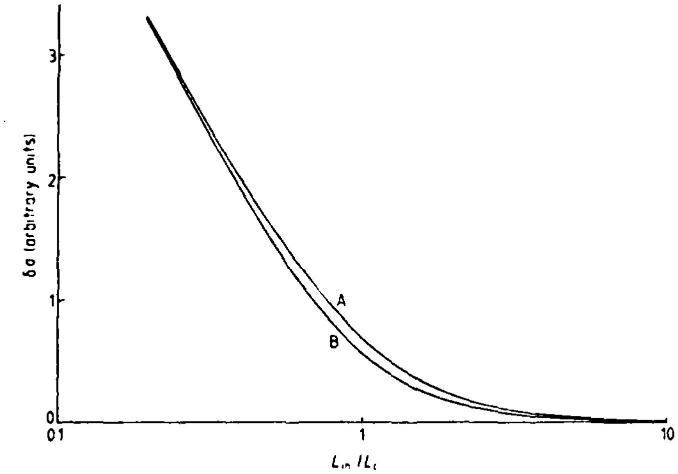

Figure 2. *Here we show the calculated change in conductivity* $\delta\sigma$ *against* L_{in}/L_c for varying L_{in}. Curve A is the graphical representation of equation (15); similarly B is equation (11) normalised to fit in the high and low L_{in} limits. In the limit $L_{in} \ll L_c$, $\delta\sigma$ according to equation (16) varies as $\ln(F/L_{in}^2)$, whereas according to equation (11) (the digamma function), this variation is $\ln(F/2L_{in}^2)$. When $L_{in} \gg L_c$, i.e. $B \to \infty$, equation (16) gives $\delta\sigma$ varying as $\ln(F/L_c^2)$, whereas the digamma function (equation (11)) gives $\delta\sigma$ varying as $\ln(2F/L_c^2) - 1.96$. The differences due to the constant factors appear to be related to the mathematical details rather than the physical processes. Our fitting procedure eliminates their effect without changing the shape. This corresponds to the real situation, as experimentally it is only the change in σ which is important. The effect of the approximation in equation (12) is that as $B \to 0$ $\delta\sigma$ varies as B rather than the more physically reasonable B^2, which comes more from the rigorous treatment of Altshuler *et al* and Hikami *et al* and was observed by Kawaguchi and Kawaji.

$$eFL_F = \hbar D/L_F^2 \tag{18}$$

as the field increases, i.e.

$$L_F = (\hbar D/eF)^{1/3}. \tag{19}$$

Thus the electric field may change $\delta\sigma$ at any temperature through a $\ln[L_d(T, F)/l]$ dependence, where

$$L_d(T, F) = (L_F^{-2} + L_{in}^{-2}(T))^{-1/2}. \tag{20}$$

Consequently, increasing F tends to delocalise the wavefunction. For large F and B we must have

$$L_d(F, B) = (L_F^{-2} + L_c^{-2}(B))^{-1/2}. \tag{21}$$

An AC field may be represented by a length $L_\omega = (D/\omega)^{1/2}$ (Gorkov *et al* 1979). We see that the AC electric field and the static magnetic field can be represented by similar length scales; these result from the period in time that the perturbation acts on the electron. In the case of an AC field it is $1/\omega$, where ω is the frequency and in the case of a magnetic field it is $1/\omega_c$, where ω_c is the cyclotron frequency. Of course, as $1/\omega_c$ leads to the length L_B it is not of great practical significance, L_B always being greater than L_c.

We now turn to the experimental verification of the universal curve. We have measured the conductivity of a (100) orientation, n-channel MOSFET at temperatures between 50 mK and 1 K in the 'metallic regime' ($k_F l \sim 4$, $R_\square = 3.4\,\mathrm{k}\Omega$). Three parameters were varied: temperature, electric field and magnetic field; and logarithmic corrections to the conductivity were observed as a function of each (Bishop *et al* 1980, Uren *et al* 1980, 1981).

For ohmic electric fields, and at a lattice temperature of ~ 50 mK, the change in conductivity $\partial\sigma$ was proportional to ln B, so that L_d was $\sim L_c$, i.e. L_{in} and $L_F \gg L_d$. We have plotted $\delta\sigma$ against L_c as crosses in figure 1. For zero magnetic field and ohmic electric fields, $\delta\sigma \propto \ln T$. Here the length was set by the inelastic scattering length $L_{in}(T)$, which in this case was due to electron–electron scattering. The inelastic length was found by use of the negative magnetoresistance described by equation (11) (Kawaguchi and Kawaji 1980, Uren *et al* 1981). Fitting this expression to the experimental data gave τ_{in} and hence L_{in}. These lengths are indicated in figure 1 as open circles

The effect of the electric field is more complicated. The observed $\delta\sigma \propto \ln F$ dependence, for zero magnetic field and low lattice temperature, may be due to field-induced heating of the electron gas to a temperature well above that of the lattice. This will result in an electric field dependent L_{in} (Anderson *et al* 1979). Alternatively, the electric field could set the new length, L_F of equation (19), directly. L_F was found to lie in the broken region of the universal curve for the electric fields used in the experiment. This implied that $L_F \gg L_{in}$ for the conditions used and so the heating mechanism was dominant. However, for lower lattice temperatures and electric fields, L_F must eventually become the shortest length. The heating model gave the electric field dependence of $L_{in}(F)$ which is plotted in figure 1 as dots.

Excellent agreement was found between the conductivities set by L_{in} and L_c, both lying on a common curve. This is good evidence that the concept of the shortest length is correct and that the proposed lengths correspond to the shortest lengths in the system. Details of the transition between these lengths will be published shortly.

Thus, to summarise we find that the diffusion corrections to the conductivity of a two-dimensional disordered system are universally given by equation (9), where L_d may depend on five variables and is given by

$$L_d = (L^{-2} + L_{in}^{-2} + L_\omega^{-2} + L_F^{-2} + L_c^{-2})^{-1/2}. \tag{22}$$

In our experiment we succeeded in verifying equation (22) for two length scales L_{in} and L_c.

We thank Professor Sir Nevill Mott for many discussions on this topic. M Kaveh thanks the SRC for a Visiting Fellowship; R A Davies and M J Uren possess SRC Research Studentships. The low-temperature measurements were performed at the SRC Ruther-

ford Laboratory and we are most grateful for the help and advice of Dr S F J Read and Mr G Regan. This work was supported by the SRC and in part by the European Research Office of the US Army.

References

Abrahams E, Anderson P W, Licciardello D C and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **42** 673
Altshuler B L, Khmelnitzkii D, Larkin A I and Lee P A 1980 *Phys. Rev.* **B22** 5142
Anderson P W, Abrahams E and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **44** 1288
Bishop D J, Tsui D C and Dynes R C 1980 *Phys. Rev. Lett.* **44** 1153
Gorkov L P, Larkin A I and Khmelnitzkii D 1979 *JETP Lett.* **30** 229
Haydock R 1981 *Phil. Mag.* **B43** 203
Hikami S, Larkin A I and Nagaoka Y 1980 *Prog. Theor. Phys.* **63** 707
Houghton A, Jericki A, Kenway R D and Pruisken A M M 1980 *Phys. Rev. Lett.* **45** 394
Kaveh M and Mott N F 1981a *J. Phys. C: Solid State Phys.* **14** L177
—— 1981b *J. Phys. C: Solid State Phys.* **14** L183
—— 1981c to be submitted
Kawaguchi Y and Kawaji S 1980 *J. Phys. Soc. Japan* **48** 699
Uren M J, Davies R A, Kaveh M and Pepper M 1981 *J. Phys. C: Solid State Phys.* **14** L395
Uren M J, Davies R A and Pepper M 1980 *J. Phys. C: Solid State Phys.* **13** L985

J. Phys. C: Solid State Phys., 14 (1981) L395–L402. Printed in Great Britain

LETTER TO THE EDITOR

Magnetic delocalisation of a two-dimensional electron gas and the quantum law of electron–electron scattering

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Abstract. We discuss the effect of a magnetic field on the weak localisation of a two-dimensional electron gas. It is shown that due to quantum corrections the electron–electron relaxation time τ_{ee} varies with electron temperature T as $\tau_{ee}^{-1} = A_1T + A_2T^2$, in the temperature range 3 K–0.1 K. This short τ_{ee} causes a rapid transition between states which are weakly localised and so reduces the logarithmic correction to the conductance.

Negative magnetoresistance in the Si inversion layer was first found by Eisele and Dorda (1974). Subsequently Pollitt *et al* (1976) showed that the magnetoresistance became positive as soon as the Fermi level passed below the mobility edge and conduction was by hopping. Kawaguchi *et al* (1978) found that the negative magnetoresistance depended only on the normal component of the magnetic field, a result which excludes spin effects as the cause of the behaviour.

It has been suggested that as a consequence of diffusion, i.e. elastic scattering, all states are weakly localised in two dimensions (Abrahams *et al* 1979, Gorkov *et al* 1979, Houghton *et al* 1980, Haydock 1981, Kaveh and Mott 1981a, b). At finite temperatures the weak (power law) localisation will produce a correction to the conductance $\Delta\sigma$, where

$$\Delta\sigma = (-2\alpha e^2/\pi^2\hbar)\ \ln\ (L/l) \tag{1}$$

L is the inelastic diffusion length and l is the elastic mean free path, α is a constant and the factor of 2 arises from the valley degeneracy. This law is found below ~ 1 K (Bishop *et al* 1980, Uren *et al* 1980). Hikami *et al* (1980) and Altshuler *et al* (1980a) have shown that the effect of a magnetic field is to alter the length scale in equation (1). The correction becomes

$$\delta\sigma = \frac{\alpha e^2}{\pi^2\hbar}\left[\psi\left(\tfrac{1}{2} + \frac{\hbar}{4eB\tau_{in}D}\right) + \ln\left(\frac{4eBD\tau}{\hbar}\right)\right] \tag{2}$$

where ψ is the digamma function, τ_{in} is the inelastic scattering time and D is the diffusion coefficient of electrons at the Fermi level; we assume a valley degeneracy of 2. The length scale is now set by both the temperature-dependent inelastic length L and the

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temperature-independent cyclotron radius L_{cyc}. When $L \ll L_{cyc}$ the magnetic field has little influence on the temperature dependence of the logarithmic correction. Decreasing L_{cyc} modifies this until, when $L_{cyc} \ll L$, the temperature dependence is lost, although the negative magnetoresistance remains. We have observed this behaviour and will report it in more detail later.

Kawaguchi and Kawaji (1980a, b) have shown that the values of τ_{in} extracted from equation (2) vary as T^P above 2 K, where $P = 2$ at high values of carrier concentration and decreases at lower values. This suggests that, as expected, the inelastic scattering was electron–electron type. The result implies that even if the logarithmic correction is not observed due to the dominance of other scattering mechanisms (Cham and Wheeler 1980, Stern 1980), equation (2) is still found as the other mechanisms are insensitive to small values of magnetic field.

In an earlier Letter (Uren *et al* 1980) we showed that a magnetic field suppressed the logarithmic term arising from the localisation and also enhanced another logarithmic mechanism. This was thought to be the Coulomb interaction model of Altshuler *et al* (1980a). However, our discussion of the screening in the treatment of Altshuler *et al* (1980a, b) was incorrect, and we now suggest that this mechanism is not Coulomb in origin. It is, however, closely related to that of Altshuler *et al*, except that the energy shift is produced by the effect of the magnetic field on the spin distribution. Further details will be published later.

In the devices used for this work l was sufficiently great that the localisation could be entirely suppressed by the magnetic field without observation of the second mechanism. In this Letter we present results on the electron–electron scattering for $k_F l \sim 4$, and show that the electron–electron scattering rate is modified by impurity scattering. Furthermore, the deviation from the normal T^{-2} behaviour persists down to ~0.1 K, and the loss of temperature dependence below ~2 K found by Kawaguchi and Kawaji (1980b) was probably due to electron heating in that experiment.

Our experiments were carried out on (100) silicon MOSFETS (silicon gate) in the temperature range 0.1–4 K. The samples were 250 × 250 μm with conventional Hall probes spaced at $\frac{1}{4}$ and $\frac{3}{4}$ of the channel length. All the resistance measurements were four-terminal, using low-frequency AC, although there was no obvious contact resistance in the specimens used.

In the range 0.1–0.6 K the magnetoresistance was measured in a dilution refrigerator with magnetic fields between 0 and 0.25 T. Above this field, the interaction regime is entered. In figure 1 the change in resistance ΔR $(R(B) - R(B = 0))$ is plotted against log B for two different electric fields (labelled A and B), the lattice temperature being maintained at 50 mK.

Fitting equation (2) gives the full curves in figure 1 showing reasonable agreement with experiment. In the previous work of Bishop *et al* and Uren *et al*, the valley degeneracy was ignored. However, it seems clear that it should be included and we have done so here. For the valley degeneracy of 2 this gave $\alpha = 0.28 \pm 0.02$, which did not appear to alter with electric field or temperature as was found by Kawaguchi and Kawaji (1980a). α is a constant which should be $\frac{1}{2}$ or 1 depending on the spin flip scattering length, so the low values found experimentally are still unexplained.

The effect of increasing the electric field is to increase the electron temperature above that of the lattice (Anderson *et al* 1979). By comparing a change in resistance against T with the change against F, the electron temperature corresponding to a particular field can be extracted (Fang and Fowler 1970). This analysis should be true provided the electron–electron scattering time is shorter than the electron–phonon

relaxation time, which the observed dependence of electron temperature on electric field shows to be the case. Furthermore, this procedure is valid regardless of the particular mechanism dominating the conductance. The values of τ_{ee} for each electron temperature were found from fitting equation (2) to the experimental data and are plotted in figure 2. In the temperature range 0.1–0.6 K the heating was accomplished by the electric field and the values of τ_{ee} are 2×10^{-10}–2×10^{-11} s. In the range 1 K–3 K the

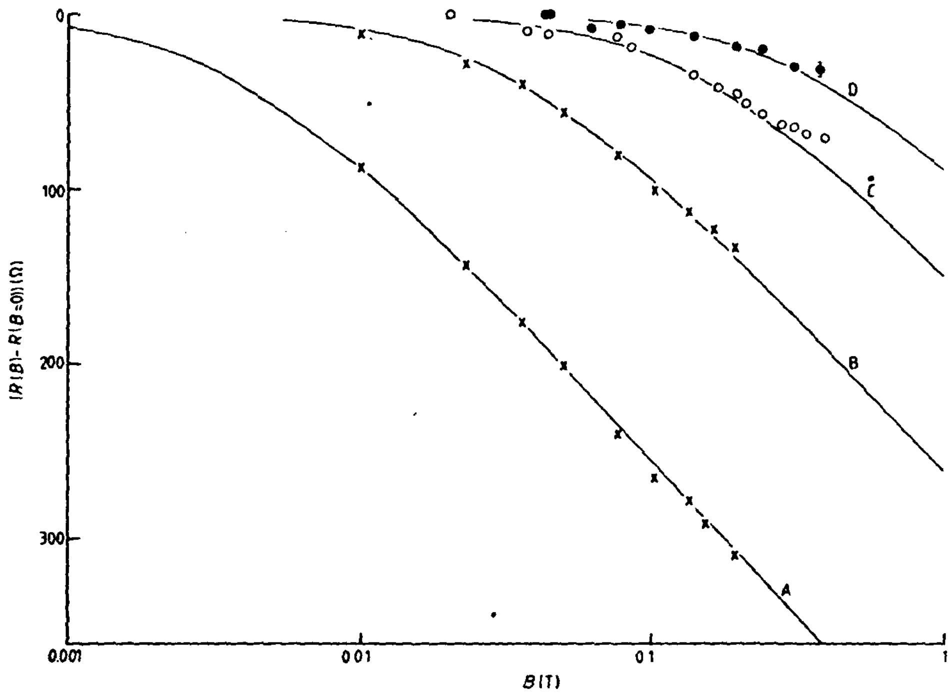

Figure 1. The change in resistance is plotted against log B. The curves are theoretical plots for $\alpha = 0.28$. A and B are for a lattice temperature of ~50 mK and electric fields of 0.2 and 2 V m^{-1} corresponding to electron temperatures of 110 ± 10 mK and 550 ± 50 mK. C and D are for ohmic electric field and temperature of 1.2 K and 2.1 K. Metallic resistance is 3.4 kΩ and N_{inv} is 3.8×10^{15} m^{-2}.

magnetoresistance was measured in the ohmic regime and the values of τ_{ee} are (5–1) $\times 10^{-12}$ s. The magnetoresistance effect vanished at about 4 K when $\tau \sim \tau_{in}$ ($\tau = 5.7 \times 10^{-13}$ s for the value of E_F used in these experiments), which is expected as the logarithmic correction should disappear when this equality occurs. Good agreement is obtained between the two temperature regimes considering the large systematic errors which can easily occur in the determination of τ_{in}.

Any fitting procedure will tend to produce artificially smooth sets of results. The error bars represent the maximum ranges of smooth lines which could be drawn during the fit of electric field to temperature and the fitting of equation (2) to the magnetoresistance. Additional errors can easily arise at long scattering times because of residual magnetic fields. The data fit a power law of $\tau_{in}^{-1} = T^{1.6}$ fairly well, but no physical reason to expect this exponent exists.

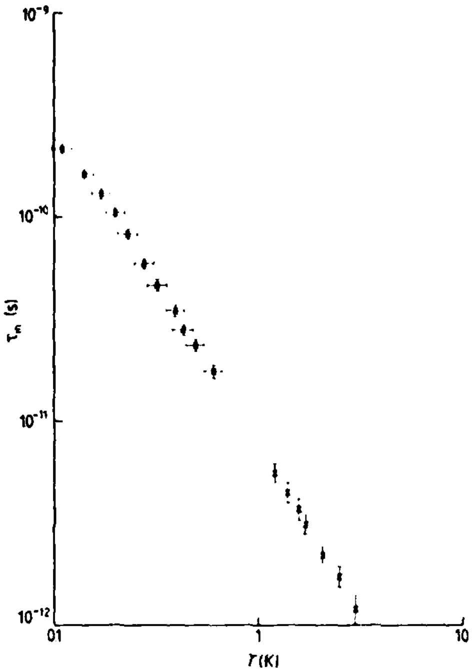

Figure 2. The log of τ_{in} extracted from equation (2) is plotted against the log of the electron temperature. In the temperature range 0.1–0.6 K the heating was accomplished using an electric field, and in the region 1.2 K–3 K the lattice temperature was changed with ohmic electric fields.

According to Landau $\tau_{ee}^{-1} \propto \varepsilon^2$, where ε is the energy of a state measured relative to E_F. From the energy dependence of the Fermi distribution of electrons it is clear that $\varepsilon \sim k_B T$, and thus $\tau_{ee}^{-1} \propto T^2$. This argument takes into account only the energy conservation of the scattering event. The electron can gain energy kT but another electron must lose energy kT; therefore the number of available events must be proportional to T^2. This argument does not depend on the momentum transfer in the process. This will determine the strength of τ_{ee} but not the temperature dependence. Recently Altshuler and Aronov (1979) and Schmid (1974) pointed out that, for samples for which $k_F l$ is small, the Landau argument must be corrected to include the finiteness of the mean free path. This is particularly important when the mean free path is short, i.e. $l \sim K_F^{-1}$. The small momentum transfer must then be correlated to the energy transfer and the ε^2 law is modified. For three dimensions Altshuler and Aronov and Schmid obtained a correction proportional to $\varepsilon^{3/2}$. Extending their ideas to two dimensions, one obtains a 'correction' proportional to ε. Thus in two dimensions, for small values of ε, the 'correction' term will determine the electron damping time. Since for a finite temperature $\varepsilon \sim k_B T$, we find $\tau_{ee}^{-1} = A_1 T + A_2 T^2$. Our estimations for A_1 and A_2 show that $T \simeq 1$ K both are nearly equal. Therefore, for investigations of the 'logarithmic region' of the conductivity τ_{ee}^{-1} should tend to a T law.

Our main argument for the quantum corrections to τ_{ee}^{-1} differs from the previously quoted work and is closely related to the argument of Kaveh and Mott (1981a, b) which yielded the same quantum corrections to the density of states (in every dimension), as

obtained by Altshuler and Aronov (1979) by diagrammatic techniques. (A two-dimensional T law has also been suggested by Abrahams *et al* (unpublished, quoted in Altshuler *et al* 1980a)). An electron in state $\boldsymbol{K}$ is scattered to state $\boldsymbol{K}_2$ by losing (or gaining) energy $\hbar\omega$. Another electron in state $\boldsymbol{K}_3$ must jump to state $\boldsymbol{K}_4$ by acquiring (or losing) the same amount of energy. The scattering time depends on all available ($\boldsymbol{K}_2$, $\boldsymbol{K}_3$, $\boldsymbol{K}_4$) states. However, conservation of momentum requires that if $\boldsymbol{K}_2 - \boldsymbol{K}_1 = \boldsymbol{q}$ then $\boldsymbol{K}_4 - \boldsymbol{K}_3 = -\boldsymbol{q}$. Conservation of energy gives $E(\boldsymbol{K}_2) - E(\boldsymbol{K}_1) = E(\boldsymbol{K}_4) - E(\boldsymbol{K}_3) = \hbar\omega$. This yields to a restriction to the possible independent scattering events. In three dimensions we are left with five independent variables to describe all possible scattering events for an electron in state $\boldsymbol{K}_1$. It is convenient to choose two energy variables and one momentum vector. The five variables are (ω, ε_3, $\boldsymbol{q}$): $\hbar\omega$ is the energy exchange in the scattering, ε_3 is the energy of state $\boldsymbol{K}_3$ of the other electron and $\boldsymbol{q}$ is the momentum transfer. This is demonstrated in figure 3.

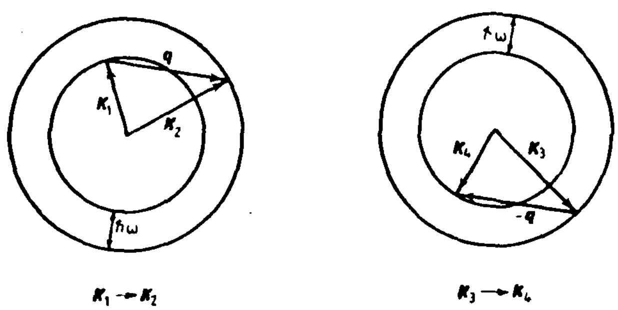

Figure 3. The diagram shows the available momentum space for each electron before and after scattering.

In two dimensions there are only three independent variables represented by (ω, ε_3, $|\boldsymbol{q}|$). The argument we present here is the same for three and two dimensions except for the differences in phase space. The electron damping time is given by

$$\tau_{ee}^{-1} \propto \int \mathrm{d}q q^{d-1} \int_0^{kT} \mathrm{d}\omega \int_0^{kT} \mathrm{d}\varepsilon_3 V_{ee}^2(q) \tag{3}$$

where d is the dimensionality. Since $\boldsymbol{K}_2$, $\boldsymbol{K}_3$, $\boldsymbol{K}_4$ can rotate only over an energy interval kT this sets an upper limit for the number of scattering events which is the upper limit of the ω and ε_3 integration.

In the usual Landau argument there is no correlation between q and the energy of a state; the q integration is just a q phase space integration yielding a number which is not dependent on temperature. Thus $\tau_{ee}^{-1} \propto T^2$ in both three and two dimensions (for one dimension see a discussion by Kaveh 1980).

The new point here is that for low transfer of momentum such that $q < l^{-1}$ the individual energy levels are broadened. In this case the momentum q is related to an energy scale which defines the energy broadening of a state near E_F. We now divide equation (3) into two contributions; the first corresponds to an integration over $l^{-1} < q < 2K_F$. This will yield the usual Landau result $\tau_{ee}^{-1} \propto T^2$. The second contribution comes from the q integration for $q < l^{-1}$. The energy broadening of a state due to elastic scattering of an electron due to the disorder is $\hbar Dq^2$. This determines a lower value of

q, since we must have

$$\hbar Dq^2 \geqslant k_B T. \tag{4}$$

Otherwise the broadening energy due to the impurity scattering is not important on the scale of the energy transferred in an electron–electron scattering of order kT. We therefore require the contribution to τ_{ee}^{-1} due to scattering events for which

$$(k_B T/\hbar D)^{1/2} < q < l^{-1}. \tag{5}$$

However, an important change must be made to equation (3) in the region of momentum exchange given by equation (5). This is seen by a similar argument to that presented by Kaveh and Mott (1981b) in their discussion of the change of the density of states in a disordered metal. There the energy broadening $\hbar Dq^2$ entered only once for a particular energy state. Here, in calculating τ_{ee}^{-1}, we must include the energy broadening twice: once for the transition for the K_1 electron and the second time for the transition of the K_3 electron (see figure 3). Equation (3) turns out to be

$$\tau_{ee}^{-1} \propto (kT)^2 \int_{(kT/\hbar D)^{\frac{1}{2}}}^{l^{-1}} \mathrm{d}q q^{d-1} \left(\frac{V_{ee}(q)}{\hbar Dq^2}\right)\left(\frac{V_{ee}(q)}{\hbar Dq^2}\right). \tag{6}$$

The q integration is the same as in the calculation of the change of density of states due to diffusion broadening (Kaveh and Mott 1981b), except of course that $V_{ee}(q)/\hbar Dq^2$ appears twice. It is always the lower limit which dominates in equation (6) and correspondingly the Landau T^2 is modified. V_{ee} is independent of q for such small values of q and is $4\pi e^2/K^2$ for $d = 3$ and $2\pi e^2/K$ for $d = 2$, where K is the inverse screening length. Hence, for $d = 3$, we obtain

$$\tau_{ee}^{-1} \propto \frac{(k_B T)^2}{(\hbar D)^2} \int_{(kT/\hbar D)^{\frac{1}{2}}}^{l^{-1}} \mathrm{d}q/q^2 \propto \left(\frac{k_B T}{\hbar D}\right)^{3/2} \tag{7}$$

which is the result first obtained by Schmid (1974) by diagrammatic techniques. Our method of obtaining equation (6) enables us to obtain easily the correction to τ_{ee}^{-1} for the two-dimensional case. For $d = 2$ we find from equation (6)

$$\tau_{ee}^{-1} \propto \frac{(kT)^2}{(\hbar D)^2} \int_{(kT/\hbar D)^{\frac{1}{2}}}^{l^{-1}} \mathrm{d}q/q^3 \propto \frac{kT}{\hbar D}. \tag{8}$$

Thus in two dimensions the $q < l^{-1}$ region is more pronounced than in three dimensions. For $T \to 0$, τ_{ee}^{-1} should tend to a T law and the Landau result will dominate only for higher temperatures. Thus for two dimensions the effect of impurity scattering is to produce a new damping law for an electron state:

$$\tau_{ee}^{-1} = A_1 T + A_2 T^2. \tag{9}$$

The origin of the quantum correction to τ_{ee}^{-1} is similar to the corrections one finds in the density of states of a two-dimensional disordered metal. In inversion layers the logarithmic correction due to electron–electron interactions is difficult to observe because of the presence of localisation effects. However, when the localisation effects are suppressed the interaction effect on the density of states is revealed (Uren *et al* 1980). Therefore the existence of a T term in τ_{ee}^{-1} is consistent with a logarithmic correction in the density of states. The same follows for three dimensions. An observation of a $T^{1/2}$ dependence in the conductivity must correspond to a $T^{3/2}$ correction in τ_{ee}^{-1}.

We accordingly looked for electron–electron scattering of this form and have

replotted the results of figure 2 in figure 4 in the form of τ_{in}^{-1}/T against T so as to separate these terms. Reasonable agreement is found with values of $A_1 = (3.4 \pm 0.6) \times 10^{10}\ \mathrm{K^{-1}\ s^{-1}}$ and $A_2 = (8.6 \pm 1.3) \times 10^{10}\ \mathrm{K^{-2}\ s^{-1}}$, which is c rtainly consistent with a T law. The T law of electron–electron scattering is enh n . at low $k_F l$

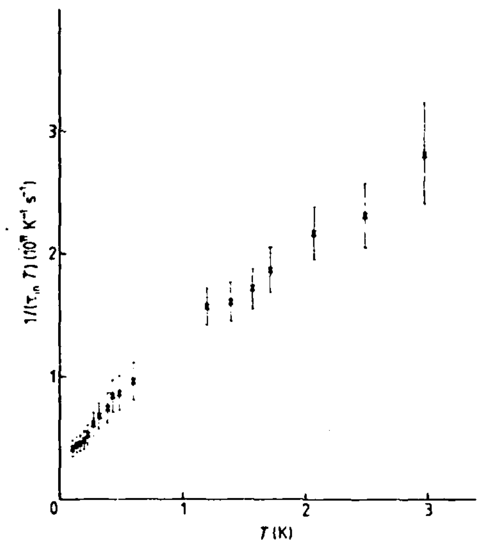

Figure 4. $1/(\tau_{in}T)$ is plotted against T. The straight line plot which does not go through the origin indicates a $\tau_{in}^{-1} = A_1T + A_2T^2$ law.

(our value is only 4), which is why it is observed here. These results and the use of the negative magnetic resistance in determining a universal length scale will be discussed in detail shortly (Kaveh *et al* to be published).

Thus, in summary, the negative magnetoresistance in (100) MOSFETs over the temperature range 0.1–4 K has been fitted to the theories of Hikami *et al* (1980) and Altshuler *et al* (1980b) and the inelastic scattering time has been extracted. This scattering time varies as $\tau_{in}^{-1} = A_1T + A_2T^2$ and is identified as electron–electron scattering where quantum corrections lead to a transition to a T law at low temperatures.

We thank Professor Sir Nevill Mott for many discussions on this topic. M Kaveh thanks the SRC for a Visiting Fellowship; R A Davies and M J Uren possess SRC Research Studentships. The low-temperature measurements were performed at the SRC Rutherford Laboratory and we are most grateful for the help and advice of Dr S F J Read and Mr G Regan. This work was supported by the SRC and in part by the European Research Office of the US Army.

References

Abrahams E, Anderson P W, Licciardello D C and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **42** 673
Altshuler B L and Aronov A G 1979 *Sov. Phys. JETP* **50** 968
Altshuler B L, Aronov A G and Lee P A 1980a *Phys. Rev. Lett.* **44** 1288

Altshuler B L, Khmelnitzkii D, Larkin A I and Lee P A 1980b *Phys. Rev.* **B22** 5142
Anderson P W, Abrahams E and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **42** 673
Bishop D J, Tsui D C and Dynes R C 1980 *Phys. Rev. Lett.* **44** 1153
Cham K M and Wheeler R G 1980 *Phys. Rev. Lett.* **44** 1472
Eisele and Dorda 1974 *Phys. Rev. Lett.* **32** 1360
Fang F and Fowler A B 1970 *J. Appl. Phys.* **41** 1825
Gorkov L P, Larkin A H and Khmelnitzkii D 1979 *JETP Lett.* **30** 229
Haydock R V 1981 *Phil. Mag.* **B43** 203
Houghton A, Jericki A, Kenway R D and Pruisken A M M 1980 *Phys. Rev. Lett.* **45** 394
Hikami S, Larkin A I and Nagaoka Y 1980 *Prog. Theor. Phys.* **63** 707
Kaveh M 1980 *J. Phys. C: Solid State Phys.* **13** L611
Kaveh M and Mott N F 1981a *J. Phys. C: Solid State Phys.* **14** L177
—— 1981b *J. Phys. C: Solid State Phys.* **14** L183
Kaveh M, Uren M J, Davies R O and Pepper M 1981 *J. Phys. C: Solid State Phys* **14**
Kawaguchi Y and Kawaji S 1980a *J. Phys. Soc. Japan* **48** 699
—— 1980b *J. Phys. Soc. Japan* **49** 983
Kawaguchi Y, Kitahara H and Kawaji S 1978 *Solid State Commun.* **26** 701
Pollitt S, Pepper M and Adkins C J 1976 *Surf. Sci.* **58** 79
Schmid A 1974 *Z. Phys.* **271** 251
Stern F 1980 *Phys. Rev. Lett.* **44** 1469
Uren M J, Davies R A and Pepper M 1980 *J. Phys. C: Solid State Phys.* **13** L985

J. Phys. C: Solid State Phys., 14 (1981) L531–L537. Printed in Great Britain

LETTER TO THE EDITOR

Magnetic separation of localisation and interaction effects in a two-dimensional electron gas at low temperatures

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Abstract. We show that by the application of a magnetic field it is possible to achieve complete separation of localisation and interaction mechanisms in two dimensions. Measurements of the conductance of silicon inversion layers show that, at certain values of magnetic field, it is also possible to achieve metallic conduction near 50 mK. It is also shown that the appearance of the interaction mechanism is not strongly dependent on the direction of the magnetic field, implying that the origin of the effect is in electron spin rather than cyclotron orbit motion. The implications for the quantised Hall resistor are discussed.

It is now well known that the conductance of a two-dimensional metal decreases logarithmically with temperature at low temperatures (Dolan and Osheroff 1979, Bishop *et al* 1980, 1981, Uren *et al* 1980, hereafter denoted UDP). In our earlier Letter, we showed that there are two distinct mechanisms giving rise to logarithmic corrections to the conductance, and that a transition between the two may be achieved by the application of a magnetic field B.

For small or zero B, carrier concentration $\leqslant 10^{17}\,\mathrm{m}^{-2}$, and $K_F l > 1$ (K_F is the Fermi wavevector, l is the mean free path); in silicon inversion layers the dominant mechanism causing the logarithmic correction is the weak localisation always present in disordered two-dimensional systems (Abrahams *et al* 1979, Gorkov *et al* 1979, Kaveh and Mott 1981a). Increasing the magnetic field suppresses this effect by making the temperature-independent, cyclotron orbit the shortest length scale (Hikami *et al* 1980, Altshuler *et al* 1980b, UDP, Kaveh *et al* 1981, Uren *et al* 1981).

Increasing the magnetic field also causes a second mechanism to become observable: UDP identified this as the interaction mechanism of Altshuler *et al* (1980a) (Kaveh and Mott (1981b) gave a more physical argument leading to Altshuler's result). It was not clear why the magnetic field enhanced this mechanism. According to Altshuler *et al* (1980a) the correction to the conductance due to interactions is given by

$$\Delta\sigma = \frac{e^2}{\hbar}\frac{1}{4\pi^2}(2 - 2F)\ln T \tag{1}$$

where the quantity F is due to screening and has a value of 0.93 for a typical carrier concentration of $3.8 \times 10^{15}\,\mathrm{m}^{-2}$, making the correction vanishingly small.

Recently P A Lee and T V Ramakrishan (1981 private communication) have suggested that the factor $(2 - 2F)$ is changed on application of a magnetic field to $(2 - F)$, by the change in spin polarisation, provided $g\beta B \gg kT$ (β is the Bohr magneton and g is the Landé factor, equal to 2). This could account for the results of UDP that the interaction mechanism was less stable than the localisation mechanism against an increase in temperature. It also predicts a positive magnetoresistance as well as the transition of $(2 - 2F)$ to $(2 - F)$. Evidence, in three dimensions, for this magnetic effect is provided by Rosenbaum *et al* (1981), who find agreement with the corresponding prediction independent of the magnetic field direction. In UDP corrections to the

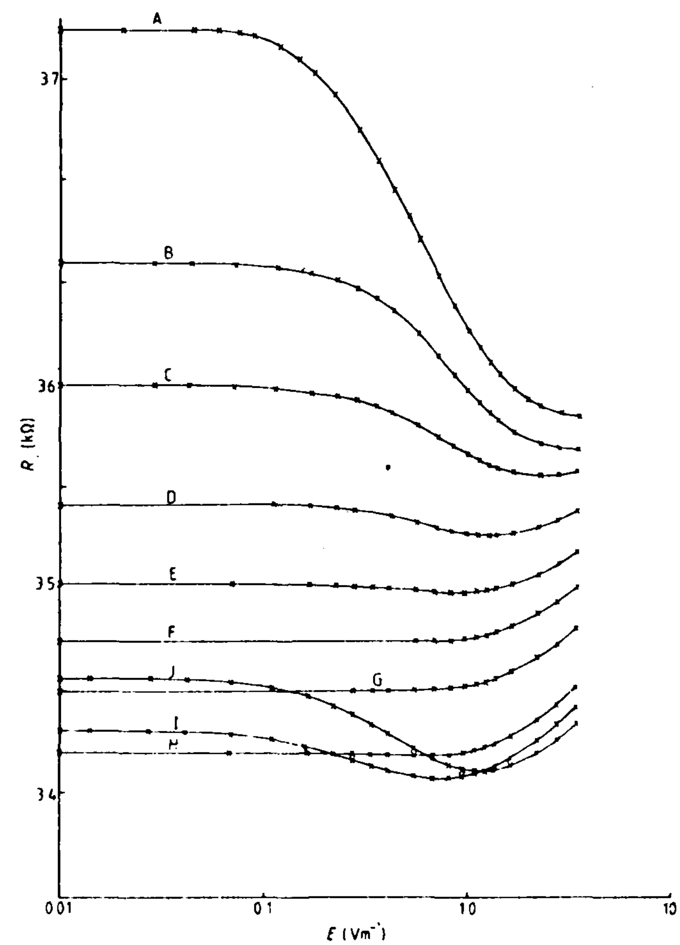

Figure 1. $R_{\square}$ is plotted against $\log E$ for various values of magnetic field with a constant carrier concentration of $3.8 \times 10^{15}\ m^{-2}$ and a lattice temperature of ~50 mK. The magnetic fields in T are: A 0; B 0.008; C 0.021; D 0.047; E 0.074; F 0.1; G 0.15; H 0.26; I 0.32; and J 0.52. The crosses mark the points used for the integration of the originally measured differential resistance. The elastic scattering length was 36 nm.

conductance were always present, the magnetic field altering the magnitude of the variation with temperature and giving magnetoresistance.

The experiments reported here were carried out on (100) orientation n-channel silicon MOSFETS, low temperatures being given by a dilution refrigerator. Details of sample geometry and measurement methods are described in UDP.

Here, using samples with a longer l than in UDP, we show that it is possible to eliminate entirely corrections to the conductance in a certain range of magnetic field. The small degree of disorder in these samples meant that the $K_F l = 1$ condition was achieved with a carrier concentration of $2.2 \times 10^{15}\ \mathrm{m}^{-2}$. In figure 1 we show the variation of inversion layer resistance with electric field for various values of B; the effect of increasing the electric field is to increase the electron temperature and hence decrease the resistance. It is seen that when $B = 0$ the resistance is initially ohmic and then, as heating occurs, decreases logarithmically, increasing again for electric fields $>1\ \mathrm{Vm}^{-1}$. This final increase is probably due to the temperature dependence of screening decreasing l (Cham and Wheeler 1980, Stern 1980) and will not be discussed further.

Increasing B produces the expected negative magnetoresistance and a decrease in the change in resistance with electron temperature. This results from the shortening of the scale length determining the conductance correction

$$\Delta\sigma \approx (-e/\pi^2 h)\ \ln(L_d/L)$$

where L_d is given by

$$\frac{1}{L_d} = \left(\frac{1}{L_{\mathrm{cyc}}^2} + \frac{1}{L_{\mathrm{IN}}^2}\right)^{1/2}.$$

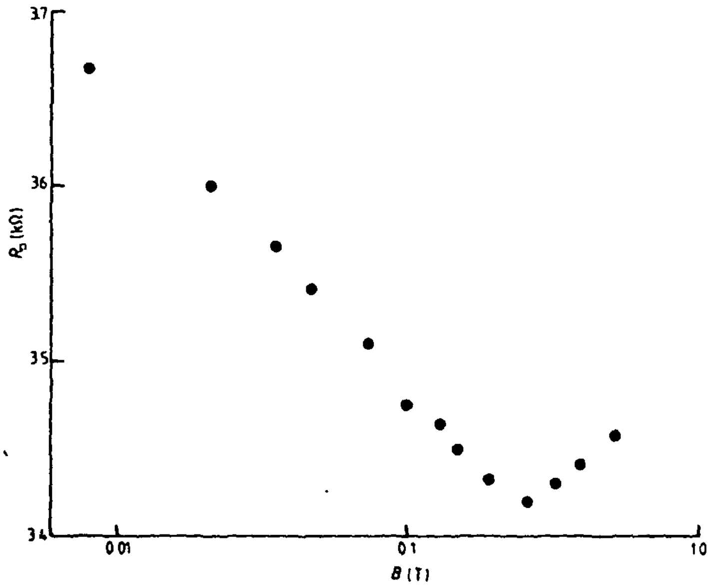

Figure 2. $R_{\square}$ is plotted against log B, for small electric field. Conditions are as in figure 1.

L_{cyc} is the cyclotron orbit and L_{IN} the inelastic length (Kaveh *et al* 1981, Uren *et al* 1981). This approximation is in good agreement with experiment and the more rigorous theory (Hikami *et al* 1980, Altshuler *et al* 1980b) except for $L_{cyc} > L_{IN}$ (small B) where it fails to capture the B^2 dependence of the negative magnetoresistance. It is also apparent that as B increases the resistance is ohmic to higher electric fields, i.e. as the cyclotron length decreases, it is necessary to heat the electrons more for L_{IN} to become less than L_{cyc}.

When $B > 0.1$ T no electric field, and hence temperature, dependence is observable, as L_{cyc} determines L_d for all accessible values of temperature; the negative magnetoresistance is still present as L_d is shortened by increasing B. Increasing B above 0.25 T results in the observation of the interaction mechanism. The localisation mechanism is still present as negative magnetoresistance is seen at high electron temperatures; this will disappear only when $L_{cyc} \leq l$, i.e. $\omega_c\tau \geq 1$ (ω_c is the cyclotron frequency and τ is the scattering time). At higher values of B ($\omega_c\tau \sim 0.25$) the positive magnetoresistance of the interaction mechanism is observed. The 50 mK magnetoresistance, at small electric fields, is shown in figure 2; it can be seen to deviate at 0.26 T. This gives a value of $g\beta B/k = 350$ mK, in agreement with the prediction that $g\beta B > kT$ to lift the spin degeneracy, making the interaction effect visible.

Compared with the results in UDP, the magnitude of the interaction effect ($\Delta R/R$

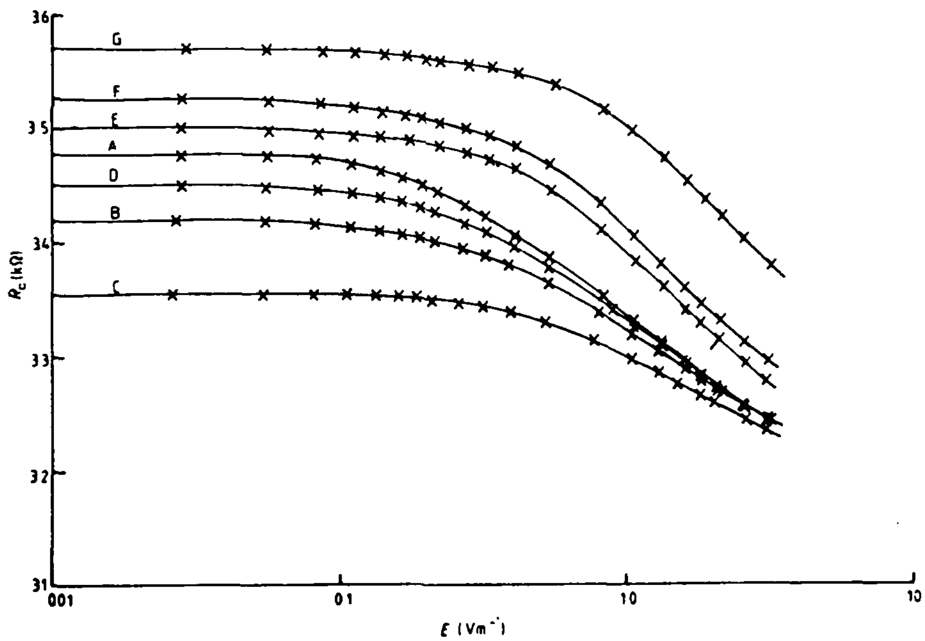

Figure 3. $R_{\square}$ is plotted against log E for various 'parallel' magnetic fields. The lattice temperature is 85 mK, carrier concentration is 1.2×10^{16} m^{-2}, and elastic scattering length 19 nm. The magnetic fields in T are: A 0; B 0.013; C 0.052; D 0.13; E 0.92; F 1.3; and G 2.6. Crosses again represent points used for integration.

per decade change in E) is much smaller. They should both be similar, as they are a measure of the pre-logarithmic term in eq. ation (1), F being similar in both cases. We suggest that this is a result of being close to the $\omega_c\tau = 1$ condition with the greater l. Here the theory will break down as the nature of the wavefunction changes and the condition that $k_Fl > 1$ becomes inapplicable.

We have also investigated corrections to the conductivity with the magnetic field parallel to the inversion layer. A sample with short l, as in UDP, was again used; $k_Fl = 3.7$ and $l = 19$ nm for the results shown.

Resistance as a function of electric field was again measured for a variety of magnetic fields, electron heating relating this to the temperature dependence. Some typical results are shown in figure 3. Figure 4 shows the magnetoresistance, at 85 mK, and also magnetoresistance for the same sample with the field perpendicular to the surface, for comparison. The two sets of results were taken on different occasions, so parameters differ

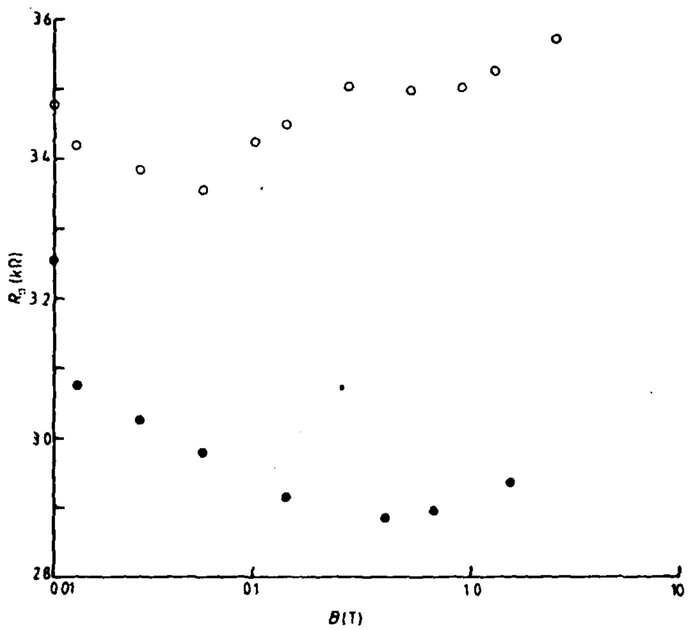

Figure 4. $R_\square$ is plotted against log B for two orientations of the field. Open symbols are for 'parallel' field (conditions as in figure 3); full symbols are for transverse field. temperature 85 mK and elastic scattering length is 17 nm.

slightly. A small amount of negative magnetoresistance is seen (with parallel B field) for $B < 0.05$ T; we attribute this to a small misalignment of the sample. Comparison with the transverse field case suggests that the field is in fact at an angle of 6° to the inversion layer. The small angle results in a slight suppression of the localisation mechanism and an initial negative magnetoresistance. Kawaguchi *et al* (1978) have shown that the negative magnetoresistance is only dependent on the transverse component.

Of greater interest is the behaviour at higher magnetic fields where in both cases positive magnetoresistance appears; this is the onset of the second interaction mechanism. With a 'parallel' magnetic field this effect is seen for smaller B, showing it to depend on the total field and not just the transverse component. It is seen at smaller B as the negative magnetoresistance, which must be exceeded, is much smaller in this case.

This absence of a directional dependence in the onset of the interaction mechanism is clear evidence for it being a spin rather than an orbital effect. In future work, the parallel field will allow a more detailed investigation of the interaction regime as B can be increased further without the $\omega_c\tau = 1$ limitation.

Although we have given an experimental demonstration that, for certain magnetic fields, temperature-independent (i.e. metallic) conduction can be obtained at temperatures as low as 50 mK, it is possible that an interaction correction will appear at still lower temperatures when $g\beta B > kT$. However, as when a magnetic field destroys the localisation, the conductance change due to interactions is only a correction term (Kaveh and Mott 1981b), the metallic conductance should be restored at even lower temperatures.

We note that the two mechanisms for logarithmic corrections are not applicable to the quantised Hall resistor (von Klitzing *et al* 1980). Here the condition is that $\omega\tau \gg 1$, and so clearly the localisation mechanism is completely suppressed. In addition the 'plateau' of constant Hall resistance is found when σ_{xx} is zero, or vanishingly small, and the current in the x direction is determined by σ_{xy}. The factor of 2 found for the ratio in the change of Hall constant to change of resistance (Uren *et al* 1980, Bishop *et al* 1981) is proof that σ_{xy} is not affected by the interaction mechanism. As the concept of $k_F l$ is inapplicable for the values of $\omega\tau$ used, the mechanism should be invalid, as indicated by the observed decrease in logarithmic gradient when $\omega\tau$ approaches unity.

Finally, we mention that the action of the magnetic field distinguishes between the interactions studied here and the two types of localisation, exponential and power law. For the case of exponential localisation the magnetic field shrinks the wavefunction, thereby producing a positive magnetoresistance, localisation increases and conduction by excitation to E_c and hopping are affected in different ways (Pepper 1979). On the other hand, the conductance when states are power law localised is increased by the magnetic field shortening the length scale. In the same way that the field sharpens the distinction between conduction by excitation to E_c and hopping in exponential localisation, so the distinction between the transport mechanisms for the two different types of localisation will be sharpened.

We have enjoyed many discussions with Professor Sir Nevill Mott and Dr M Kaveh. M J Uren and R A Davies receive SRC studentships. These experiments were carried out at the SRC Rutherford Laboratory and we are grateful for the help and advice of Mr G Regan and Dr S F J Read. This work was supported by SRC and, in part, by the European Research Office of the US Army.

Note added in proof. Recently Lee and Ramakrishan have suggested that, in 2D, if $g\beta B \gg kT$ the magnetoresistance in the interaction regime varies as $(e^2/\hbar)(F/4\pi^2)\ln B$. We confirm this law to a reasonable accuracy (final points of figure 4). Poole, Pepper and Glew have recently investigated localisation and interaction effects in 2D transport in modulation-doped GaAs. Here, due to the smaller F, both the logarithmic corrections are present at zero magnetic field. However, the localisation can be suppressed by a small value of B and the interaction effects remain, giving the Hall ratio of 2.

References

Abrahams E, Anderson P W, Licciardello D C and Ramakrishan T V 1979 *Phys. Rev. Lett.* **42** 673
Altshuler B L, Aronov A G and Lee P A 1980a *Phys. Rev. Lett.* **44** 1288
Altshuler B L, Khmelñitzkii C, Larkin A I and Lee P A 1980b *Phys. Rev.* **B22** 5142
Bishop D J, Tsui D C and Dynes R C 1980 *Phys. Rev. Lett.* **44** 1153
—— 1981 *Phys. Rev. Lett.* **46** 360
Cham K M and Wheeler R G 1980 *Phys. Rev. Lett.* **44** 1472
Dolan C J and Osheroff D D 1979 *Phys. Rev. Lett.* **43** 721
Gorkov L P, Larkin A I and Khmelñitzkii D E 1979 *JETP Lett.* **30** 229
Hikami S, Larkin A I and Nagaoka Y 1980 *Prog. Theor. Phys.* **63** 707
Kaveh M and Mott N F 1981a *J. Phys. C: Solid State Phys.* **14** L177
—— 1981b *J. Phys. C: Solid State Phys.* **14** L183
Kaveh M, Uren M J, Davies R A and Pepper M 1981 *J. Phys. C: Solid State Phys.* **14** L413
Kawaguchi Y, Kitahara H and Kawaji S 1978 *Solid State Commun.* **26** 701
von Klitzing K, Dorda G and Pepper M 1980 *Phys. Rev. Lett.* **45** 494
Pepper M 1979 *J. Non Cryst. Solids* **32** 161
Rosenbaum T F, Thomas G and Andres K 1981 *APS meeting, March*
Stern F 1980 *Phys. Rev. Lett.* **44** 1469
Uren M J, Davies R A, Kaveh M and Pepper M 1981 *J. Phys. C: Solid State Phys.* **14** L395
Uren M J, Davies R A and Pepper M 1980 *J. Phys. C: Solid State Phys.* **13** L985

J. Phys. C: Solid State Phys., 14 (1981) L995–L1005. Printed in Great Britain

LETTER TO THE EDITOR

The observation of localisation and interaction effects in the two-dimensional electron gas of a GaAs–GaAlAs heterojunction at low temperatures

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Abstract. We have investigated the logarithmic correction to the transport properties of the two-dimensional electron gas at the (modulation-doped) GaAs–GaAlAs interface in the temperature range 4.2–0.34 K. GaAs is different to Si in that, due to the low density of states, the electron screening length is greater. This allows the existence of a significant logarithmic correction from the electron–electron interaction in the absence of a magnetic field. The experimental results are consistent with the co-existence of localisation and interaction effects although the analysis is complicated by the occupation of two sub-bands.

We have investigated the electron–electron scattering rate and find that, as in the Si inversion layer, the temperature dependence is reduced by quantum corrections. Analysis of the rate of emission of phonons by hot electrons indicates that the phonons of importance are two-dimensional.

It is known that at low temperatures the conductance of a two-dimensional electron gas decreases logarithmically with falling temperature. This effect has been seen in silicon inversion layers (Bishop *et al* 1980, 1981, Uren *et al* 1980, Davies *et al* 1981, Uren *et al* 1981, Kaveh *et al* 1981). The correction to the conductance $\Delta\sigma$ due to localisation is

$$\Delta\sigma = (e^2/\pi^2\hbar)\alpha \ln T^{p/2} \tag{1}$$

where α and p are constants (Abrahams *et al* 1979, Gorkov *et al* 1979, Kaveh and Mott 1981, Pichard and Sarma 1981), and for interactions (Altshuler *et al* 1980a, b) in the presence of weak impurity scattering

$$\Delta\sigma = (e^2/4\pi^2\hbar)(2 - 2F) \ln T \tag{2}$$

where F is the electron–electron screening factor defined by

$$F = \int_0^{2\pi} \frac{\partial\theta}{2(1 + 2k_F/K)}. \tag{3}$$

k_F is the Fermi k vector and K, the 2D electron inverse screening length, is given by

$$K = m^* e^2/2\pi\varepsilon\varepsilon_0\hbar^2. \tag{4}$$

In the presence of a magnetic field B the localisation contribution $\Delta\sigma$ can be quenched

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as the cyclotron diameter becomes much less than the inelastic scattering length (Uren *et al* 1981). This leads to a negative magnetoresistance described by Hikami *et al* (1980) and Altshuler *et al* (1980a). The conductance is now given by

$$\sigma = \sigma_0 - \frac{e^2\alpha}{2\pi^2\hbar}\left[\psi\left(\tfrac{1}{2} + \frac{\hbar}{4\tau eBD}\right) - \psi\left(\tfrac{1}{2} + \frac{\hbar}{4\tau_{\mathrm{IN}}eBD}\right)\right] \tag{5}$$

where σ_0 is the normal conductance $\sigma_0 = n_e e^2\tau/m^*$, τ is the elastic scattering time, τ_{IN} is the inelastic scattering time, D is the electron diffusion constant and ψ is the digamma function. We have assumed that τ_{IN} is much shorter than τ_{so} and τ_{s}, the two-dimensional spin orbit and magnetic scattering times respectively.

The interaction mechanism is enhanced by a magnetic field which allows a separation between these two mechanisms (Uren *et al* 1980). The field converts the $(2-2F)$ term to $(2-F)$, so enhancing the logarithmic corrections, provided $g\beta B > kT$ (Lee and Ramakrishnan 1981). This results in a positive magnetoresistance as has been observed by Davies *et al* (1981) in silicon inversion layers.

Altshuler *et al* (1980a, b) calculated the effect of interactions on σ_{xy} and obtained $\Delta\sigma_{xy} = 0$. Thus

$$\Delta R_{\mathrm{H}}/R_{\mathrm{H}} = -2\Delta\sigma_{xx}/\sigma_{xx}. \tag{6}$$

where R_{H} is the Hall constant. However, in the localisation regime the Hall mobility varies with temperature in the same manner as the conductance (Fukuyama 1980), i.e.

$$\Delta R_{\mathrm{H}}/R_{\mathrm{H}} = 0. \tag{7}$$

Thus the application of a magnetic field allows the identification of the localisation and interaction contributions to $\Delta\sigma$. Furthermore, as has been shown by Davies *et al* (1981), the localisation is affected by the transverse component of the field, whereas the enhancement of the interaction effect is independent of the direction of the field.

In the previous work on Si inversion layer the interaction component was small in the absence of a magnetic field as F was near unity. The greater screening length in GaAs results in a much smaller value of F for similar values of carrier concentration. We therefore investigated this regime of behaviour in modulation-doped GaAs structures.

If highly doped GaAlAs is grown onto undoped GaAs then, due to the higher electron affinity of GaAs, electrons spill over from the GaAlAs into a narrow potential well as illustrated in figure 1. Since the electrons are spatially removed from their parent donor ions then impurity scattering is reduced, resulting in high electron mobilities μ_e at low temperatures (Dingle *et al* 1978), values of $\mu_e \sim 211\,000\ \mathrm{cm^2\,V^{-1}\,s^{-1}}$ at 10 K having been reported (Drummond *et al* 1981). The degenerate electron gas is quantised (Störmer *et al* 1979, Tsui and Logan 1979) and at low temperatures transport is two-dimensional. The 2D electron gas will be referred to as a 2DEG. Our sample was grown by liquid phase epitaxy into a multilayer structure with seven alternate layers of GaAs and 8 of $Ga_{1-x}Al_xAs$ ($x = 0.3$), all ~3500 Å thick, to form essentially 14 2DEG layers in parallel. The substrate was semi-insulating, Cr-doped GaAs. The sample was of four-terminal Van der Pauw 'clover leaf' geometry with indium contacts annealed in an H_2 atmosphere at 400°C. It was necessary to diffuse in the contacts to a sufficient depth to ensure that all the 2DEG layers were electrically connected in parallel. All measurements made were four-terminal, using low-frequency AC. The magnetic field was reversed during both Hall and magnetoresistance measurements and an average taken to cancel out any unwanted offset voltages. Following the usual Van der Pauw method, results from all possible contact configurations were averaged.

We measured Shubnikov–de Haas oscillations in the resistance at magnetic fields up to 15 T, the differential of resistance showing additional structure above ~2.5 T. The measurements were repeated with the sample tilted at various angles θ with respect to the magnetic field direction as shown in figure 2. The oscillations and structure were dependent only on the perpendicular component of the field showing that electron spin splitting, which is independent of field direction, was not contributing. This dependence

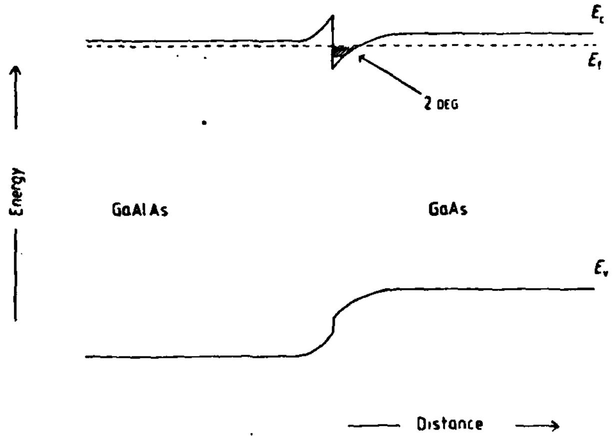

Figure 1. Schematic energy band diagram for a GaAs–GaAlAs interface showing the formation of a two-dimensional electron gas (2DEG).

further proves the 2D nature of the system and leads us to attribute the structure to the occupation of a second sub-band. Measurement of the oscillation amplitude with temperature, below 2.5 T, gave an effective mass m^* of 0.073 ± 0.003 in agreement with Störmer *et al* (1979). Plotting the oscillation periodicity against $1/B$ gave a straight line

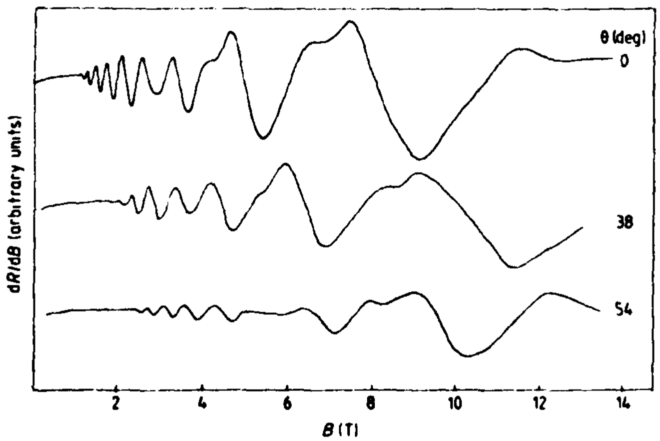

Figure 2. Shubnikov–de Haas oscillations in the differential of resistance are shown for three angles of sample tilt θ with respect to the magnetic field direction. The oscillations are dependent only on the perpendicular component of magnetic field indicating that two sub-bands are occupied.

relationship below ~2.5 T, as shown in figure 3, corresponding to a value of Fermi energy E_F of ~14.5 meV. A frequency analysis confirmed that two sub-bands were occupied with energies ~26 meV (lower sub-band) and ~14.5 meV (upper sub-band) as shown in the inset of figure 3. This corresponds to an electron concentration n of 8.8×10^{11} cm^{-2} and 4.9×10^{11} cm^{-2} for the lower and upper sub-band respectively using

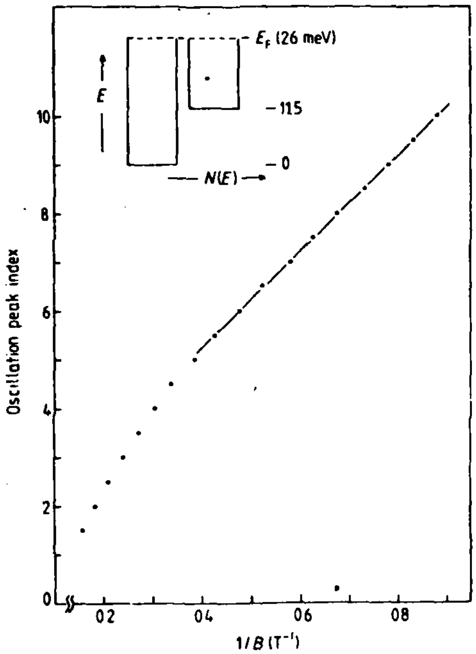

Figure 3. The oscillation peak index from figure 2 ($\theta = 0°$) is plotted against $1/B$. The straight line corresponds to a value of $E_F \sim 14.5$ meV, the deviation at high fields being due to the lower electron mobility sub-band of energy ~26 meV. The peak index is similar to the Landau level index with a phase change introduced due to the differentiation of the resistance. The inset shows a schematic energy diagram of the 2DEG sub-bands believed to be contributing to the conductance.

our value of $m = 0.073\, m_0$. The lower sub-band has a much lower electron mobility μ_l than the upper sub-band, and thus it is not observed until $B \gtrsim 2$ T and has a reduced amplitude. The higher mobility in the sub-band is possibly a consequence of the form of the wavefunction resulting in decreased scattering at the interface.

Measurement of the low-field Hall constant R_H initially gave a value of 98; however, after reannealing the device contacts at a later stage R_H fell to 63 with a corresponding decrease in sheet resistance R. We concluded that initially only nine of the 2DEG layers were contributing to the conductance and only after the further contact diffusion did all 14 layers contribute. All results presented assume nine 2DEG layers, with 2 independent

sub-bands per layer whose contribution to the change in resistance $\Delta R/R$ is given by

$$\frac{\Delta R}{R} = \frac{\Delta R_1 R_2^2 + \Delta R_2 R_1^2}{R_1 R_2 (R_1 + R_2)} \tag{8}$$

$$(\Delta R = -R^2 \Delta\sigma)$$

i.e. the two sub-bands conduct in parallel with each other. The subscripts 1 and 2 refer to the lower and upper sub-bands respectively.

For the two-band Hall effect, R_H is given by

$$R_H = \frac{1}{n_1 e} \frac{1 + n_2/n_1 (\mu_2/\mu_1)^2}{(1 + (n_2/n_1)\mu_2/\mu_1)^2} \tag{9}$$

and

$$\frac{1}{R} = n_1 e \mu_1 + n_2 e \mu_2. \tag{10}$$

Combining equations (9) and (10) gives $\mu_1 \sim 1800\ \mathrm{cm^2\,V\,s^{-1}}$ and $\mu_2 \sim 14500\ \mathrm{cm^2\,V\,s^{-1}}$ corresponding to values of $k_F l$ of ~6 and ~28 where l is the elastic scattering length.

The density of states, which has been assumed to be the same in both sub-bands is $3.4 \times 10^{10}\ \mathrm{e^-/cm^2\,meV}$, and is lower than in the ground sub-band of (100) orientated Si ($1.7 \times 10^{11}\ \mathrm{e^-/cm^2\,meV}$) because of the single valley degeneracy and the lower m^* in GaAs. This means that the inverse screening length K is smaller, leading to values of the screening factor F from equation (3) of 0.5 and 0.57 for the lower and upper sub-bands respectively; thus the $\Delta\sigma$ due to interactions will be significant (even at low magnetic fields when $g\beta B < kT$) as predicted by equation (2). This is in contrast to the silicon inversion layer where typically $F = 0.9$, and interaction effects are very small in the absence of a magnetic field.

The remainder of this paper is mainly devoted to the localisation and interaction effects. In order to interpret the results we adopted the following procedure, the necessary assumptions being justified later in the paper.

(i) The measured negative magnetoresistance was assumed to be due only to localisation in the higher-mobility upper sub-band. This led to values of α, p, τ_{IN} and hence the contribution to the total resistance change ΔR per decade of temperature T for this sub-band was obtained.

(ii) The contribution to ΔR per decade of T from interactions in both sub-bands was calculated using equation (2). This was experimentally obtained by measurement of the change in R_H with T and using equation (6). The validity of this procedure is only correct if Fukuyamas' prediction (equation (7)) is correct. The quantity $g\mu_B B$ was much less than kT in our case, and so the magnetic field will not affect the interaction process.

(iii) The remaining contribution to ΔR per decade of T was assumed to be due to localisation in the lower sub-band. The product αp was thus obtained for this band.

In figure 4 the sample resistance per 2DEG layer is plotted against the logarithm of temperature, from 340 mK to 4.2 K and as a function of B. For $B = 0$ a straight line through the experimental points gives $\Delta R/R \sim 6.6\%$ per decade of temperature. Application of a perpendicular magnetic field, <0.1 T, resulted in a negative magnetoresistance arising from the quenching of the localisation contribution to ΔR, as shown in figure 5. This effect is lost with field applied parallel. Increasing B much above 0.2 T resulted in a positive magnetoresistance and then Shubnikov–de Haas oscillations as

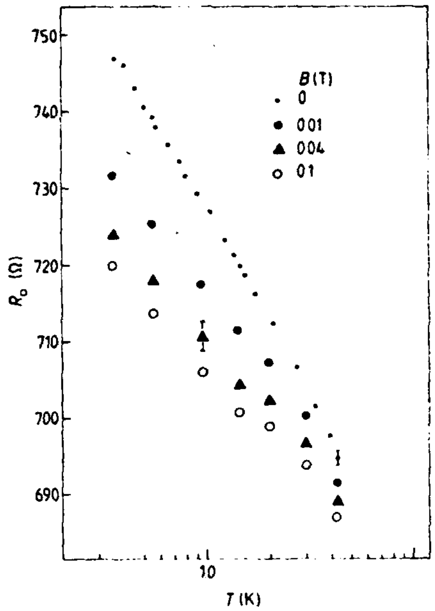

Figure 4. The resistance per 2DEG layer is plotted against the log of the temperature as a function of perpendicular magnetic field. The points for $B = 0.1$ T were extracted from the extrapolated regions (dotted curve) of figure 5.

Landau levels were formed at ~0.5 T (just less than $\omega_c\tau \sim 1$). Clearly, both sub-bands will behave differently in a magnetic field since the diffusivity D is a factor of ~4 higher for the upper sub-band; also, the inelastic scattering time τ_{IN} will be greater owing to the lower electron concentration. Thus, according to equation (5), the upper sub-band will

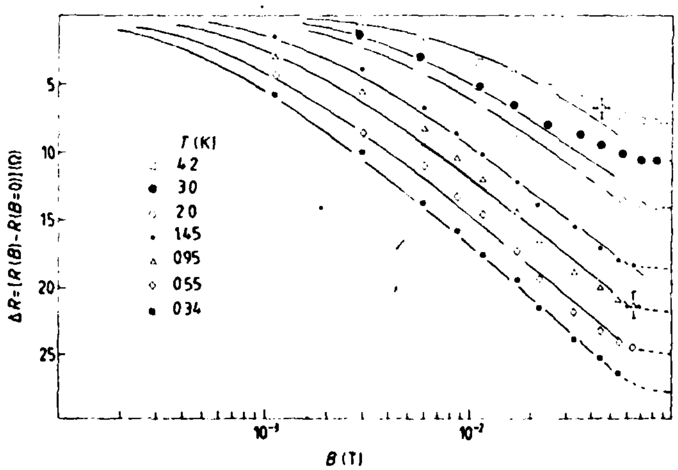

Figure 5. The change in resistance per 2DEG layer is plotted against magnetic field as a function of temperature. The dotted lines are extrapolations to $B = 0.1$ T. The full curves are theoretical plots for $\alpha = 0.82$ using equation (5).

show negative magnetoresistance, as its localisation is quenched, before the lower sub-band. To simplify the analysis we have neglected the contribution from the lower sub-band up to 0.1 T and assumed that at 0.1 T, where a deviation from equation (5) is found, all the upper sub-band localisation has been quenched. At higher fields Landau levels are formed in the upper sub-band as Shubnikov–de Haas oscillations dominate the conductivity, preventing the observation of the negative magnetoresistance contribution from the lower sub-band. A value of $ap \sim 1.0$ for the upper sub-band was obtained from the $B = 0.1$ T data points of figure 4 using equations (1) and (8) with $\Delta R_1 = 0$. The full curves in figure 5 were obtained by fitting equation (5) with $\alpha = 0.82$ although a temperature-dependent α above 2 K was evident, the reason for this being unknown at present. This allows values of τ_{IN} to be obtained for each temperature, these being plotted in figure 6. A straight line through these points gives $\tau_{IN} \approx T^{-1/2}$, i.e. the

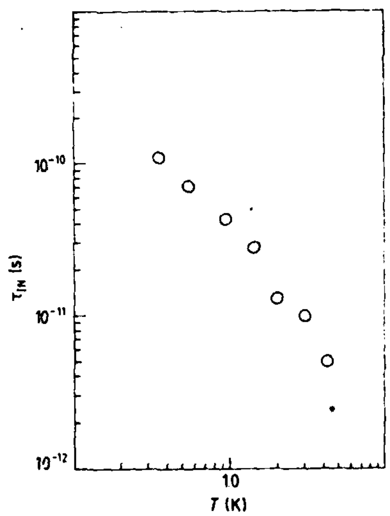

Figure 6. The inelastic scattering time is plotted against the log of the temperature, the values having been extracted from the theoretical curves of figure 5. A straight line through the points gives $\tau_{IN} \propto T^{-1.2}$.

constant $p \sim 1.2$ and $ap \sim 1.0$. This is in agreement with the results of Uren *et al* (1981), who also found that quantum corrections lowered the power of temperature from the Landau–Baber value of 2. Our value of ap is the same as that obtained from figure 4 ($B = 0.1$ T), showing that the approximation $\Delta R_1 = 0$ is reasonable.

The Hall constant R_H was measured via the integration of dV_H/dB as a function of B and T. At a constant temperature R_H was found to be constant over the range 0.02 to 0.10 T (the maximum field measured) to within the accuracy of measurement of 0.5%, as would be expected from Fukuyama's prediction (equation (7)). This of course assumes that $\Delta\sigma_{xx}$ due to interactions is constant over this range, which should be the case since $g\beta B$ is always less than kT ($g \approx 0.52$ for GaAs: Ando and Uemura 1974). Below 0.02 T, structure (illustrated in figure 7) in R_H was seen which was strongly temperature dependent, being zero at 4.2 K and increasing to as much as a 15% change in R_H at 340 mK.

The origin of this effect is unknown, although since it persisted with magnetic field parallel it possibly arises from an electron spin or a 3D contact effect. Similar structure was observed in a second sample investigated.

Because of the structure the temperature dependence of R_H was measured in the regime where the structure was not apparent. R_H was found to increase with decreasing temperature and is plotted as $\Delta R_H/R_H$ in figure 8 along with $\Delta R/R$ $(B=0)$ and the calculated change in resistance $\Delta R^{int}/R$ due to interactions only. All the plots are normalised to $T=4.2$ K; a value of $\Delta R^{int}/R$ of 1.9% per decade of temperature was obtained from equations (2) and (8) using our calculated values of F of 0.57 and 0.5. It

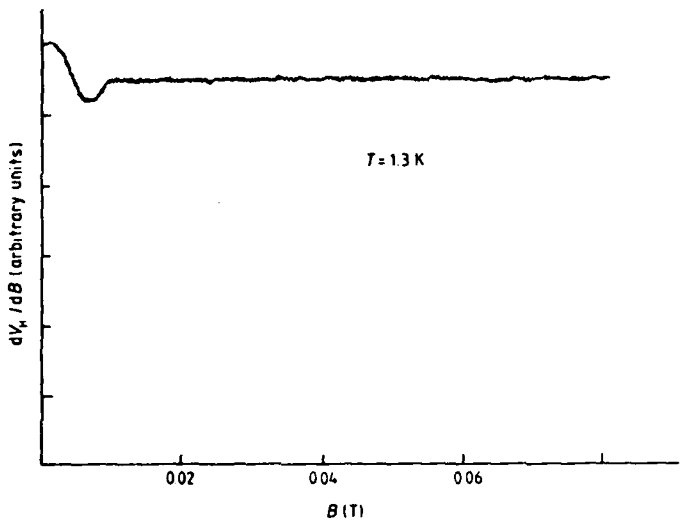

Figure 7. A typical chart recording of the differential Hall voltage against magnetic field illustrating structure below 0.02 T.

can be seen that $\Delta R_H/R_H$ is approximately $2\,\Delta R^{int}/R$ as predicted by equation (6*a*). The product $ap \sim 1.0$ obtained from the negative magnetoresistance similarly gives the change in resistance contribution of 2.2% per decade of temperature for localisation, from the upper sub-band $\Delta R_{loc}/R$, from equation (1). The total change in resistance, $(\Delta R/R)$ $(B=0) \approx 6.6\%$ per decade of temperature, should be equal to the sum of $\Delta R^{int}/R$ and $\Delta R_{loc}/R$ where the contribution from both sub-bands is included. This means that a factor of $\sim 2.5\%$ per decade of temperature should be due to localisation in the lower sub-band corresponding to $ap \sim 1.15$ obtained from equations (1) and (8). Thus, within the approximation made, ap is similar for both sub-bands.

The differential device output voltage dV/dI was measured as a function of constant current, and hence the average electric field E, at a lattice temperature of 340 mK. Increasing E eventually heats the electrons above the lattice temperature, leading to a decrease in resistance as the logarithmic component is reduced. The electron temperature T_e can be extracted by comparison of the change in resistance with the plot of $\Delta R/R$ against T (at $B=0$) from figure 2. T_e is found to be proportional to $E^{1/2}$, as shown

in figure 9, indicating that the electrons couple to 2D phonons (Anderson *et al* 1979). It should be noted that all previous measurements were made within the ohmic region of electric field ($E < 0.4\ \mathrm{V\ m^{-1}}$).

We neglect any significant modification to our measurements due to conduction in the GaAlAs impurity band for the following reasons.

(i) The donor ion concentration n_d of $10^{17}\ \mathrm{cm^{-3}}$ is the same as that required at the (Mott) metal–insulator transition using the equation for a 3D system

$$(\varepsilon_r/m^*)a_0 n_d^{1/3} \sim 1/3 \tag{11}$$

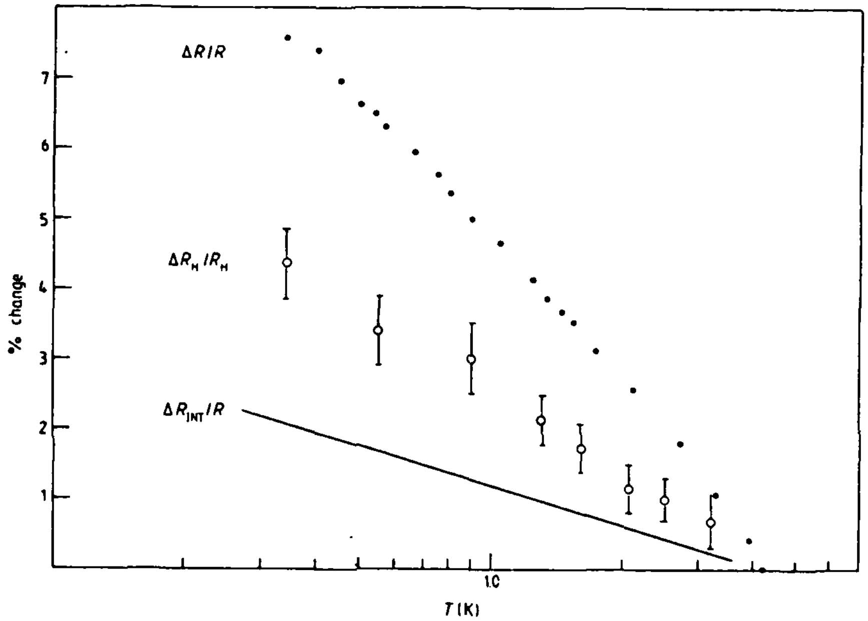

Figure 8. The percentage change in the Hall constant (measured in the region where it is independent of magnetic field) and the theoretical change in resistance due to interactions only are plotted against the log of the temperature. The total change in resistance extracted from figure 4 for $B = 0$ is also included for comparison; this is plotted in the form of dots.

where a_0 is the Bohr radius of the hydrogen atom (0.529 Å), $\varepsilon = 12.2$ and $m^* = 0.092$ for $x = 0.2$ GaAlAs (Casey and Panish 1978). Thus a metallic impurity band may not have formed. Our previous measurements on a sample of bulk GaAlAs ($n_d \approx 10^{17}\ \mathrm{cm^{-3}}$) have confirmed that conduction is activated.

(ii) A large fraction of the GaAlAs will be depleted by the adjacent 2DEG layers, further reducing the number of electrons.

In conclusion, we have observed localisation and interaction effects in GaAs which agree with current theories, although precise analysis of our results is complicated by the occupation of two sub-bands.

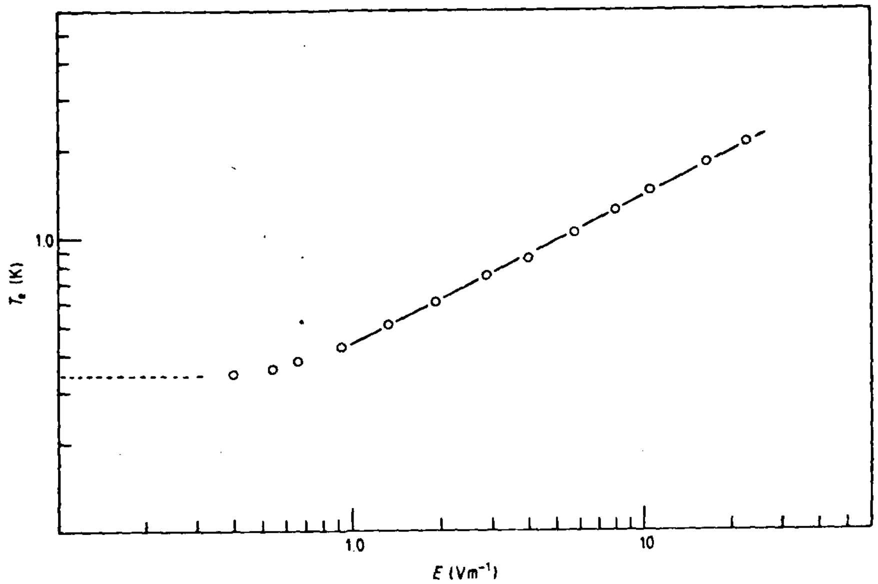

Figure 9. The log of electron temperature T_E is plotted against the log of the electric field E for a lattice temperature of 0.34 K. The straight line shows that $T_E^2 \propto E$.

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References

Abrahams E, Anderson P W, Licciardello D C and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **42** 673

Altshuler B L, Aronov A G and Lee P A 1980a *Phys. Rev. Lett.* **44** 1288

Altshuler B L, Khmelnitzkii D, Larkin A I and Lee P A 1980b *Phys. Rev.* **B22** 5142

Anderson P W, Abrahams E and Ramakrishnan T V 1979 *Phys. Rev. Lett.* **43** 718

Ando T and Uemura Y 1974 *J. Phys. Soc. Japan* **37** 1044

Bishop D J, Tsui D C and Dynes R C 1980 *Phys. Rev. Lett.* **44** 1153

—— 1981 *Phys. Rev. Lett.* **46** 360

Casey H C and Panish M B 1978 *Heterojunction Lasers* (New York: Academic Press)

Davies R A, Uren M J and Pepper M 1981 *J. Phys. C: Solid State Phys.* **19** L726

Dingle R, Störmer H L, Gossard A C and Wiegmann W 1978 *Appl. Phys. Lett.* **33** 665

Drummond T J, Kopp W and Morkoç H 1981 *Electron. Lett.* **17** 442

Fukuyama H 1980 *J. Phys. Soc. Japan* **48** 2169

Gorkov L P, Larkin A H and Khmelnitzkii D 1979 *JETP Lett.* **30** 229

Hikami S, Larkin A I and Nagaoka Y 1980 *Prog. Theor. Phys.* **63** 707

Kaveh M and Mott N F 1981 *J. Phys. C: Solid State Phys.* **14** L177

Kaveh M, Uren M J, Davies R A and Pepper M 1981 *J. Phys. C: Solid State Phys.* **14** L413

Lee P A and Ramakrishnan T V 1981 *Phys. Rev. Lett.* in press
Pichard J L and Sarma G 1981 *J. Phys. C: Solid State Phys* in the press
Stormer H L, Dingle R, Gossard A C, Wiegmann W and Sturge H D 1979 *Solid State Commun.* **29** 705
Tsui D C and Logan R A 1979 *Appl. Phys. Lett.* **35** 99
Uren M J, Davies R A, Kaveh M and Pepper M 1981 *J. Phys. C: Solid State Phys.* **14** 395
Uren M J, Davies R A and Pepper M 1980 *J. Phys. C: Solid State Phys.* **13** L985

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